The Direction of Innovation

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Abstract

How are resources allocated across different R&D areas? We show that under a plausible set of assumptions, the competitive market allocates excessive innovative efforts into high returns areas, meaning those with higher private (and social) payoffs. The underlying source of market failure is the absence of property rights on problems to be solved, which are the source of R&D value. The competitive bias towards high return areas comes three distortions. The first two are familiar in the context of a single innovation line: the cannibalization of returns of competing innovators and duplication costs. The third one is new to the innovation literature: excessive entry into high return areas results as the market does not take into account the future value of an unsolved problem while a social planner does. Allocation of resources to problem solving leads to a stationary distribution over open problems. The distribution of the socially optimal solution stochastically dominates that of the competitive equilibrium. A severe form of rent dissipation occurs in the latter, where the total value of R&D activity equals the value of allocating all resources to the least valuable problem solved.

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1 Introduction

How do competing R&D firms and researchers choose their lines of research? Will they disproportionately engage in 'hot' R&D areas and research lines? Or will they shun away from congested R&D areas, each seeking its own special niche? Dating back to at least Schumpeter (1911), there is a large literature on innovation markets, welfare assessment of R&D competition, and R&D policy design. Most research asks whether market R&D investment levels are socially optimal, and how to design policies that optimize investment decisions.[1] This paper asks a novel, important question: Are market R&D investments allocated optimally across research areas? Does R&D go in the 'right' direction? In a simple, general model, we find that it does not: R&D competition pushes firms to disproportionately engage in hot R&D lines, characterized by higher expected rates of return. As we explain later in detail, the identification of this form of market failure is an entirely novel contribution to the study of innovation markets.

Our result follows from these basic arguments. First, note that the function relating the number of researchers engaged in a R&D line with the probability of innovation discovery can be understood as a cumulative distribution function. Under a simple regularity assumption satisfied by many common statistical distributions, the marginal increment in the innovation probability increases less fast than the average probability of innovation per-researcher, when more researchers are engaged into a R&D line. Both the optimal and the competitive allocations of researchers across R&D lines are such that more engage in more promising R&D lines. The optimal allocation equalizes across R&D lines the marginal probability to achieve innovations, multiplied by these innovations' values. Instead, competing firms invest in different R&D lines so as to equalize the average probability of innovation per-researcher (again, multiplied by the innovations' values). Thus, the equilibrium allocation of researchers across R&D lines is suboptimal. Too many are engaged in the 'hot' R&D lines with higher expected rates of return, whereas too few engage in less promising research lines.[2]

[1]For a nice survey on the literature on IP policy design and enforcement, see Rockett (2010).

[2]Summarizing this line of reasoning: R&D firms who engage in R&D races do not internalize the

As anticipated earlier, this form of market failure is novel in the study of R&D.[3] Indeed, our results are derived within a model in which the social values of innovations coincide with the values appropriated by the innovators. So, the type of market inefficiency we identify is orthogonal to well understood forms of market failure that arise because of imperfect property rights of innovations returns.[4] Also, R&D firms are perfectly competitive in our model, and have access to perfect financial markets. In sum, the market inefficiency we identify cannot be ascribed to arguments in the Schumpeterian tradition.

Further, while our analysis covers also the case of sequential innovation vintages, our results are also orthogonal to the much studied inefficiencies caused by incomplete appropriability of the positive externalities borne by an innovation on follow-on innovations.[5] Likewise, and more subtly, it is important to underline that our findings hold also in models that abstract from the possibility of value complementarities or substitutabilities among different innovations or different R&D approaches towards the same innovation.[6] Importantly, this clarifies the novelty of our findings relative to earlier work on the allocation of R&D investment across R&D approaches or R&D lines, as these papers' results hinge on value complementarities or substitutabilities. Finally, we note that our analysis is staged in a framework with complete information. It is well known that asymmetric information may lead to herding, and hence we expect that it would make our results hold a fortiori.[7]

Once established our fundamental results within a simple and general model, we continue the analysis by enriching the framework, so as to build a dynamic model with canon-

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reduction in expected returns borne on other potential innovators; and while this is true in all research lines, this negative externality is larger in R&D lines with higher expected rates of return.

[3] We discuss previously identified sources of R&D market inefficiency in the literature review section 2.

[4] The social value of an innovation may be higher than its private value because of innovation externalities, technological spillovers, or imitation by competitors; and it can be lower in the case of product market displacement. Of course, if the social value of an innovation is larger than its private value, it may attract more competing firms than it is socially optimal. But it is not clear whether this difference should be larger for hot innovations, or for less promising research.

[5] Such externalities occur when the follow-on would not feasible without the first innovation, and the follow-on innovator is distinct from the first innovator.

[6] We extend our basic model to include this possibility in Appendix A, and show that our result that R&D firms overinvest in hot areas extend also to this case.

[7] Information economics studies of herding and multi-armed bandits (e.g., Banerjee, 1992, Bikhchandani et al., 1992, Bolton and Harris, 1999, Smith and Sørensen, 2000, Keller et al., 2005, Rosenberg et al. 2007, Murto and Välimäki, 2009) do not directly address R&D races, as they abstract from competition (exceptions are Moscarini and Squintani, 2010, and Halac, Kartik and Liu, 2015).

ical assumptions about innovation discovery and R&D competition. Our framework is sufficiently flexible and detailed to generate quantitative predictions, and can be taken to the data. Further, its canonical features allow an immediate comparison with many of the models in the literature on R&D. So, we can clearly identify the precise features of R&D competition that lead to our market inefficiency results.

We assume that there is a large number (ideally, a continuum) of R&D lines with a continuum of innovation values. Each innovation's discovery is an independent random event, equally likely across all engaged researchers, and with time-constant hazard rate. We first assume that arrival rates are constant across innovations, and that researchers cannot be moved across R&D lines over time (their allocation is chosen at time zero). We confirm that firms overinvest in hot R&D lines also within this 'canonical' model. To demonstrate that this framework easily leads to quantitative predictions, we calculate the equilibrium welfare loss relative to first best, stipulating a specific functional form for the innovation value distribution. Supposing that the innovation values are Pareto distributed of parameter η, we find that the welfare loss can be significant for a wide range of parameters.[8]

We then extend our canonical dynamic model to incorporate different features of interest. We lift the assumption that researchers cannot ever be moved across R&D lines, and allow for their costly redeployment at any point in time. Due to the stationarity of the problem, we find that R&D firms never move researchers away from a R&D line (nor it is ever optimal to do so), unless the research line is exhausted as a consequence of innovation discovery. So, we can approach again the problem using standard dynamic programming techniques. This allows us to recover our market failure result that R&D firms overinvest in hot R&D lines (unless moving researchers across R&D lines is perfectly costless). Importantly, we single out two separate effects through which costly switching of researchers across R&D lines lead to the market failure that is the theme of this paper.[9]

[8]The welfare loss is negligible when η is close to its lower extreme, 1, but it quickly increases as η grows, so as to reach its maximum of about 16%, for η close to 1.34, to then slowly decrease and disappear asymptotically as $\eta \to \infty$.

[9]Also, maintaining the assumption that reearchers cannot be moved across R&D lines, we allow for the possibility that the arrival rates of innovations differ across R&D lines. We recover our finding that

The analysis continues with the consideration of R&D duplicative efforts, modelled by assuming that the aggregate arrival rate of an innovation discovery grows less than linearly with the number of engaged researchers.[10] Duplication costs are a well-established fact in the R&D literature, at least since Loury (1979) and Reinganum (1982). They have been identified as a force leading to socially excessive investment in experimentation, thus acting in the opposite direction as 'classical' sources of market inefficiency (e.g., imperfect property rights of innovations returns, and imperfect access to financing). Instead, we show that duplication costs reinforce the novel form of market inefficiency that is the theme of this paper. We show that R&D firms over-invest in hot R&D lines even if researcher redeployment is perfectly costless, when allowing for R&D duplicative efforts. Again, we precisely identify the separate effects that lead to this result.

The analysis ends by returning to the case without duplicative efforts (but with costly redeployment of researchers), to introduce the possibility of successive innovation vintages. We suppose that novel profitable R&D lines arise over time and replace exhausted R&D lines that already led to the discovery of profitable innovations. We focus on R&D replacements that keep the economy in steady state. Thus, we have in mind 'mature' R&D industries in which the distribution of innovation opportunities is stationary because of the steady emergence of successive innovation vintages. Again, we recover the result that R&D firms overinvest in the hot R&D lines. Further, we show that the equilibrium and welfare functions reduce to simple closed-form expressions.

The implications of our study in terms of policy are transparent. Because competing firms overinvest in hot R&D lines in the market equilibrium, non-market based incentives that rebalance remuneration across R&D lines should be devised, so as to subsidize R&D lines with less profitable or less feasible innovations.[11] As argued below, both innovation

R&D firms overinvest in hot areas, with the obvious modification that the attractiveness of R&D lines is not determined only by the innovations' expected market values, but also by the innovations' expected feasibility.

[10]In the process of achieving a patentable innovation, competing firms often need to go through the same intermediate steps. This occurs independently of each other firms intermediate results, which are usually kept secret. As a result, the arrival rate of an innovation often increases less than linearly in the number of engaged researchers.

[11]Reasonably, subsidization should take place at an aggregate research area level. This because the overwhelming majority of individual patents have no market value whatsoever. For a humourous account of

subsidies and the up-front remuneration of ongoing research may be important, here. We discuss the literature on the subsidization of R&D in section 2, whereas in the conclusive section 5, we briefly surmise how existing funding mechanisms would plausibly fare vis-a-vis the form of market inefficiency identified in this paper. Here, it suffices to mention that these mechanisms include research grants, fiscal incentives on innovations or ongoing research, research prizes, and procurement (often for military purposes, but also for large civilian projects like public transport systems). While often State-funded, R&D subsidization can also be funded by private consortia or donors (especially, when taking the form of research grants and prizes). And the tenure system in academic institutions also entails R&D subsidization.[12] Because subsidies can be at least partially funded with levies collected on patent monopoly profits, the kind of policy intervention advocated here contains elements of cross-subsidization across R&D areas.

We conclude this introduction by underlining one important feature of the market failure uncovered in this paper: that is caused by an institutionally unavoidable missing market constraint. The welfare analysis in section 4 clarifies that the externalities exerted by R&D firms on competitors within the same R&D race could be internalized only if markets were capable to fully remunerate R&D investments and effort up-front, instead of just rewarding innovation discovery through the award of patent monopoly profit. But of course, this is impossible because of well understood impediments due to incomplete information, moral hazard, and R&D secrecy.

The paper is presented as follows. The related literature is discussed in the next section. Section 3 establishes our basic market inefficiency results within a simple general model. Section 4 enriches this framework and builds a dynamic model with canonical assumptions

the phenomenon of trivial patents, see http://images.businessweek.com/ss/09/04/0408_ridiculous_patents/ According to the then director of public affairs for the U.S. Patent & Trademark Office, "There [were] around 1.5 million patents in effect [in the U.S. in 2005], and of those, maybe 3,000 were commercially viable," see http://www.businessweek.com/stories/2005-11-09/avoiding-the-inventors-lament

[12] A sense of the size of R&D spending can be gained from the latest (2012) OECD US figures. While business enterprise spending was approximately 234,570 million dollars, equal to 59% of total R&D expediture and leading to a tax credit of 11,100 million dollars (4.7% of business expenditure), Government spending was 122,163 million dollars (31% of total R&D expenses). Of the remaining R&D spending, 11,813 million dollars came from academia, 13,092 from non-profit organizations, and 15,070 from foreign sources.

that we extend in several directions, so as to identify the precise features of R&D competition that lead to our market inefficiency results. Section 5 concludes. The least informative part of the formal analysis is in appendix.

2 Literature Review

The form of market inefficiency identified in this paper is novel. We now briefly discuss forms of market failure previously singled out in the R&D literature and policy remedies proposed to alleviate them. Later in the section, literature on the existing mechanisms for the funding of R&D is briefly reviewed.

Early literature (e.g., Schumpeter, 1911; Arrow, 1962; and Nelson, 1959) pointed at limited appropriability of the innovations' social value by innovators, and at limited access to finances as the main distorting forces in R&D markets, both leading to the implication that market investment in R&D is insufficient relative to first best. There appears to be significant empirical evidence for these forces. Since the classical work of Mansfield et al. (1977) estimates of the social return of innovations calculate that they may be twice as large as private returns to innovators; whereas evidence of a "funding gap" for investment innovation has been documented, for example, by Hall and Lerner (2010), especially in countries where public equity markets for 'venture capitalist exit' are not highly developed.

A large academic literature has developed to provide policy remedies, often advocating strong innovation protection rights,[13] and the subsidization of R&D. Wright (1983) compares patents, prizes, and procurement as three alternative mechanisms to support R&D. Patents have the advantage that they delegate R&D investment decisions to the 'informed parties,' R&D firms, and provide incentives so that they exert R&D effort efficiently. But prizes and subsidies could sometimes dominate patents, as they do not induce a monopoly welfare loss. Instead of awarding monopolistic IP through patents, the state could award a prize equal to monopoly profit, place the innovation in the public domain and obtain

[13]However, others also underline the negative effects of patents on social welfare through monopoly pricing, and on the incentives for future innovations; and Boldrin and Levine (2008) have even provocatively challenged the views that patents are needed to remunerate R&D activity.

a welfare gain. To obviate the objection that the State would hardly know the value of monopoly profit at the time of invention, Kremer (1998) suggests an ingenious mechanism based on the idea of patent buyout.[14] Relatedly, Cornelli and Schankerman (1999) show that the optimal direct mechanism to reward R&D firms whose productivity is private information can be implemented by using either an up-front menu of patent lengths and fees or a renewal fee scheme. Scotchmer (1999) finds that a similar mechanism would also be optimal when the economy has a single firm that innovates once and privately knows the R&D cost and innovation value. Hopenhayn and Mitchell (2001) show that if innovations differ both in terms of expected returns and 'fertility' (a more fertile innovation allows competitors to generate follow-ons that replace the innovation in a short time span), then the optimal contract involves a menu of patent lengths and breadths.

One source of market inefficiency that has received much attention in the literature is due to the sequential, cumulative nature of innovations. As pointed out by Horstmann, MacDonald and Slivinski (1985) and Scotchmer (1991) this 'sequential spillover' problem arises when, without a 'first' innovation, the idea for 'follow-on' innovations cannot exist, and the follow-on innovators are distinct from the first innovator. This problem is especially significant when the first innovation is 'basic research' with little market value, and the follow-on innovations are highly-profitable applications. And as well as distorting investment decisions, sequential innovations can also make the timing of innovation disclosure inefficient (see, for example, Matutes et al., 1996, and Hopenhayn and Squintani, 2016).

A large academic literature has developed to study optimal patent length and breadth for this 'cumulative innovation case' (e.g., Green and Scotchmer, 1995, Scotchmer, 1996, O'Donoghue et al., 1998, O'Donoghue, 1998, Denicoló, 2000). Summarizing their conclusions, there appears to be a strong argument for protection from literal imitation (large lagging breadth) if licensing is fully flexible and efficient, and a general strong argument

[14] After the innovator exercised full patent rights for a short period of time, the patent is sold with a second price auction. With small probability, the winner of the auction is awarded the patent; else, the innovation is placed in the public domain. In either case, the innovator is paid the highest bid in the auction as a reward for the innovation. The idea is that, at the time of the auction, the industry has had the time to form an assessment of the market value of the innovation, and this assessment can be elicited by the planner through the auction mechanism.

for leading breadth, whereas strong patentability requirements receive some support when licensing does not function well. There is also a vast literature advocating subsidization of 'basic' research, starting at least as early as Nelson (1959), by means of various mechanisms including research grants and prizes, as well as academic research funding.[15] In terms of mechanism design, Hopenhayn et al. (2006) study a quality ladder model of cumulative innovations and find the optimal mechanism to be a mandatory buyout system.[16]

The issue of innovation spillovers may complicate policy design not only among sequential innovations, but because of 'horizontal' market value complementarities or substitutabilities among innovations (see Cardon and Sasaki, 1998, and Lemley and Shapiro, 2007, for example). This possibility is entirely distinct from the market inefficiency we identify in this paper: our results hold also in the case of innovations whose market values are independent of each other.[17] A possible policy remedy to inefficiencies caused by horizontal spillovers are patent pools: agreements among patent owners to license a set of their patents to one another or to third parties. These arrangements have existed since, at least, the 1856 sewing machine pool. A recent legal doctrine requires that the patents included in the pool be complements, and that patents in the pool must not have close substitutes outside the pool, so as not to foreclose competing patents (see also Shapiro, 2001). Lerner and Tirole (2004) build a tractable model of a patent pool, and identify a simple condition to establish whether patent pools are welfare enhancing.[18]

[15]Aghion, Dewatripont, Stein (2008) provide a different account of why it can be socially optimal to have basic research be subsidized in academia. By serving as a precommitment mechanism that allows scientists to freely pursue their own interests, academia can be indispensable for early-stage research.

[16]The buyout takes the form of a transfer fee (function of innovation quality and paid to the granting authority), and a payment to the current market leader for its displacement, as well as a specified buyout amount that the new leader would accept to be displaced by another.

[17]Even further distantly related to our work, there is also a literature studying the welfare effects of complementarities and substitutabilities among different research approaches to achieve the same innovation (e.g., Bhattacharya and Mookherjee, 1986; Dasgupta and Maskin, 1987). Of course, this is very different from the analysis of this paper, which considers several innovations, without distinguishing different approaches to achieve any of them. A recent paper, Bryan and Lemus (2013), provides a valuable general model that encompasses the models cited here, as well as models of horizontal spillovers and of sequential innovation. Building on the interaction across these different kinds of spillovers, this framework can lead to important novel findings.

[18]They note that a member of the pool who considers whether to individually license its innovation may be constrained by either a 'competition margin' or a 'demand margin.' The demand margin binds if the licensor could individually raise its license price without triggering an exclusion from the patents selected by the licensees. They find that patent pools always increase welfare when the demand margin binds in

We devote the second part of this section to briefly reviewing research on existing mechanisms for the funding of R&D.

The main form of State intervention to increase the level of innovation is funding through grants and direct subsidies. Interestingly, grants are a relative modern invention: for most of history, publicly sponsored research was in-house.[19] In the 1930s, R&D funding by the U.S. federal government was a small percentage of total R&D (between 12% and 20%). This share greatly increased until 1953, to then slowly decrease to about 31% in 2012 (cf., footnote 12).[20] Extramural funding is now predominant in federally funded R&D spending (approximately 70%, in 2012).

Another important form of R&D subsidization are fiscal incentives. The U.S. Research Credit can be used to offset current, prior, and future corporate income tax liability, for 20% of R&D expenditures exceeding a 'base amount', or 14% of the excess of the qualified research expenditures over 50% of the average of the three prior years' expenditures (Alternative Simplified Credit).[21] Similar fiscal incentives are in place in Japan and France, whereas China has an even more generous incentives, including a tax deduction equal to 150% of qualifying R&D expenses, a reduced 15% corporate tax rate for companies granted 'High and New Technology Enterprise' status; as well as VAT exemption/Zero-rated treatment for certain R&D services performed for foreign entities.[22] Instead, the German tax system does not currently include relief for R&D expenditure, and R&D is funded almost entirely through grants.

While less quantitatively relevant nowadays, research prizes also play a role in stimu-

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the absence of pool, and that, as patents become more substitutable, the competition margin more likely binds and the pool more likely decreases welfare.

[19]As reported by Maurer and Scotchmer (2004), this dates back at least to ancient Egypt, where the engineer Imhotep was hired for the building of pyramids. Among the ranks of scientists working at courts over the centuries, one can list, for example, Archimedes, Kepler, Brahe, Euler, and Lagrange.

[20]The latest OECD reported share of R&D spending paid by the State is similar in other leading economies as China (21%, in 2013), Japan (17%, in 2013), Germany (29%, in 2012), France (35%, in 2012), and the UK (27%, in 2012).

[21]There are also special credits for basic research (e.g., research conducted in universities), payments to energy research consortium, and research relating to orphan drug.

[22]Likewise, the UK offers the 225% super deduction for R&D expenses for companies with less than 500 employees, with cash credits for up to 24.75% of the qualifying expenditure, and 130% super deduction for larger companies, with 10% taxable credit, for larger companies. These and the above figures are reported in the "2014 Global Survey of R&D Tax Incentives," published by Deloitte T.T. Ltd.

lating R&D activity.[23] For example, aerospace research has been fostered partly by the 'X Prize Foundation' established in 1996 with a $10 million prize for the first private firm to carry three passengers to a suborbital height of 100 km twice within a fortnight. The development of CFC-free refrigerators was pushed by the 'Super Efficient Refrigerator Prize' of $30m announced in 1992, the first initiative of the "Golden Carrot" awards sponsored by the U.S. Environmental Protection Agency, in partnership with non-profit companies, utilities and environmental groups.[24] One further form of R&D State funding is contract research, or procurement. This mechanism is most common for military innovations, and often takes the form of prototype competition.[25]

One of the most common type of research funding paid up-front takes place in the universities.[26] Academia's distinctive advantage as a R&D institution is that it motivates scientists by allowing them to freely pursue their own research interests. These motivations have been recently complemented with market-based incentives, through legislation such as the Bayh-Dole and Stevenson-Wydler Acts of 1980, which authorize the patenting of innovations developed with federal funds in universities and national laboratories, and the Federal Technology Transfer Act of 1986, that lets State universities and national laboratories create partnerships with private entities (called CRADAs), and thus spin off their innovations into the private sector. These reforms led to substantial expansion of patents for university performed research (see, for example, Lach and Schankerman, 2004), and to the relative increase of industry support for university research.[27]

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[23] Earlier R&D prizes include the award offered by the French Empire in 1795 for a means to preserve food to feed Napoleon's armies and navy, and awarded in 1810 to Nicolas Appert. (His technique, based on heat-sterilization of food packed in bottles, is still in use). Other targeted prizes led to improvements of the steam engine, to the first water turbine, and to the precise determination of longitude on ships.

[24] The contest was won by Whirlpool. But the prize was not paid because sales fell about 30% short of target required in the SERP call.

[25] For example, in the 1970's the US Air Force adopted a system, which led to the F-16 and F-18 fighter jets, where two rival companies received contracts to build prototypes followed by a flight competition to demonstrate quality. Earlier, the Air Force usually acquired prototypes from a single vendor after a contest to choose the best written proposal; that procedure was abandoned as it led to large cost overruns in the 1960's (see, Maurer and Scotchmer, 2004).

[26] Scholarly research by tenured academics has a long history, dating back to the Library of Alexandria, which supported resident scholars such as Archimedes, Hipparchos, Eratosthenes, Euclid, and Hero of Alexandria. Inventors as famous as Galileo, Newton, Dalton, Volta, Ampere and Maxwell all worked as university residents, instead of in response to project-specific incentives.

[27] Jaffe (1989) finds a significant effect of university research on corporate patents through 'geographical spillovers', particularly in the areas of Drugs and Medical Technology, and Electronics, Optics, and Nuclear

The economic assessment of the mechanisms of R&D funding we have described depends on whether or not they efficiently succeed in stimulating R&D that would have not otherwise taken place. The evidence is not always favorable. For example, Wallsten (2000) estimates a simultaneous model of expenditure and funding for a sample of U.S. firms recipient of grants through the Small Business Innovation Research program. Controlling for the endogeneity of grants, he finds no evidence of R&D investment increase and evidence of full crowding-out of private investment. In contrast, Lach (2002) estimates the relative increase in R&D expenditures of subsidized versus nonsubsidized firms using panel data on a sample of Israeli companies, and finds a positive effect for small firms, that only fades away in the largest firms. Likewise, González, Jaumandreu and Pazó (2005) structurally estimate the effects of R&D subsidies on the firms' (extensive) decisions about whether or not to perform R&D, in a dataset of Spanish manufacturing firms. They find no evidence of crowding out of private funds. Subsidies appear to stimulate R&D, and some firms would stop R&D activity in their absence. Similarly, Bloom, Griffith and Van Reenen (2000) estimate a model of R&D investment using a panel of data on tax changes and R&D spending in nine OECD countries over a 19-year period (1979–1997). They find evidence that tax incentives are effective in increasing R&D intensity, even after allowing for permanent country-specific characteristics, world macro shocks and other policy influences.[28] To conclude this brief review, we mention recent work by Takalo, Tanayama and Toivanen (2013).[29] To assess welfare effects of targeted R&D subsidies, they formulate a model that includes the application and R&D investment decisions of firms and the subsidy-granting decision of the public granting agency. Their structural estimation, based on unusually detailed project-level data from Finland, assesses the social rate of return on targeted subsidies between 30% to 50%.

Technology. A study by Mansfield (1995) on data from 66 firms in 7 manufacturing industries and from over 200 academic researchers finds that a substantial proportion of innovations in high-technology industries have been based directly on recent academic research.

[28] Specifically, they estimate that a 10% fall in the cost of R&D stimulates slightly over a 1% rise in the level of R&D in the short-run, and just under a 10% rise in R&D in the long-run.

[29] The literature on the economic assessment of R&D subsidization is very large; for reasons of space, our review is by necessity incomplete.

3 A Simple Model

There are two research lines $j = 1, 2$, with one potential innovation each. Upon discovery, innovation j delivers value z_j to the successful innovator. To isolate the findings of this paper from the well-known effects discussed earlier, we assume that the social values of innovations coincide with the private values z_1 and z_2. Without loss of generality, we assume that $z_2 > z_1 (> 0)$: the 'hot R&D line' is research line 2.

A continuum of $M > 0$ researchers inelastically supplied, needs to be allocated across the R&D lines. When a mass of researchers m_j participates in the R&D race on innovation j, the discovery of j occurs with probability $P(m_j)$. We assume that the function P is twice differentiable, strictly increasing and concave (i.e., that $P' > 0$ and $P'' < 0$), with $P(0) = 0$. When more researchers are engaged in an R&D line, the overall probability of discovery increases, but there are diminishing returns. Note that these assumptions imply that $P(m)/m$ strictly decreases in m.[30]

For either R&D line j, each individual researcher's expected value of engaging in the R&D line j is $z_j P(m_j)/m_j$, when m_j researchers are engaged in line j, because each one of them is equally likely to win the R&D race. To simplify the exposition, we assume that the two innovations $j = 1, 2$ enter additively in the social welfare function W, so that $W(m_1, m_2) = z_1 P(m_1) + z_2 P(m_2)$.[31]

We focus on interior equilibria (m_1, m_2), so that $0 < \{m_1, m_2\} < M$. It is immediate that an interior allocation (m_1, m_2) is an equilibrium if and only if $m_1 + m_2 = M$ and the following 'no-arbitrage condition' holds:

$$z_1 P(m_1)/m_1 = z_2 P(m_2)/m_2. \tag{1}$$

In an interior equilibrium (m_1, m_2), the expected private value of researching line j is the same across $j = 1, 2$.[32] Further, because $P(m)/m$ decreases in m, it must be the case that

[30] In fact, the derivative of $P(m)/m$ is proportional to $P'(m)m - P(m)$ which is strictly smaller than zero because P is strictly concave and $P(0) = 0$.

[31] This assumption is not needed to show our results here (Proposition 1, stated below). In appendix A, we generalize Proposition 1 to any utility and welfare specification.

[32] An interior equilibrium exists as long as z_1/z_2 is not too small relative to $P(M)/M$. A sufficient condition is that $\lim_{m \downarrow 0} P(m)/m = \infty$.

$m_1 < m_2$ in equilibrium.

In order to assess the equilibrium welfare properties, we now determine the optimal allocation. A social planner chooses $\tilde{m}_1$ and $\tilde{m}_2$ so as to maximize the welfare function $W(\tilde{m}_1, \tilde{m}_2) = z_1 P(\tilde{m}_1) + z_2 P(\tilde{m}_2)$ subject to the condition $\tilde{m}_1 + \tilde{m}_2 = M$. Equating the first-order conditions, at an interior solution:

$$z_1 P'(\tilde{m}_1) = z_2 P'(\tilde{m}_2). \tag{2}$$

Because P is strictly concave, it follows that $\tilde{m}_1 < \tilde{m}_2$.

The comparison of the equilibrium and social planner solutions hinges on the following regularity condition on the discovery probability function $P(\cdot)$. First, note that, because $P(m)$ increases in m and equals to zero for $m = 0$, it can be interpreted as a cumulative distribution function. Hence, the function $\Gamma(m) = mP'(m)/P(m)$ is the *generalized reverse hazard rate* of the distribution P. The key regularity condition we impose is that $\Gamma(m)$ strictly decreases in m.[33] As shown by the online appendix of Che, Dessein and Kartik (2013), this is a weak assumption. It is satisfied by many common statistical distributions with support in the positive reals (e.g., the exponential distribution, the Pareto distribution, the power function distribution, the Weibull distribution, and the Gamma distribution of parameter bigger than 1).

Under this condition, we show that firms over-invest in the hot R&D line 2, relative to the optimal solution $(\tilde{m}_1, \tilde{m}_2)$. In other terms, we prove that $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.

Proposition 1 *Suppose that the innovation discovery probability $P(m)$ is increasing and concave, and the associated generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m. In equilibrium, R&D firms overinvest in the hot R&D line 2, relative to the optimal allocation of researchers; i.e., $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.*

Proof. Dividing the equilibrium no-arbitrage condition (1) by the optimal solution condition (2), we obtain:

$$\frac{P(m_1)/m_1}{P'(\tilde{m}_1)} = \frac{P(m_2)/m_2}{P'(\tilde{m}_2)}. \tag{3}$$

[33] Note that the earlier assumption that $P(m)/m$ decreases in m implies that $\partial P(m)/\partial m \propto mP'(m) - P(m) \leq 0$, so that $\Gamma(m) = mP'(m)/P(m) \leq 1$.

Now suppose by contradiction that $m_2 \leq \tilde{m}_2$, so that $m_1 \geq \tilde{m}_1$. Then, using the concavity of P and the assumption that the associated generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m, we obtain:

$$\frac{P(m_2)/m_2}{P'(\tilde{m}_2)} > \frac{P(m_2)/m_2}{P'(m_2)} > \frac{P(m_1)/m_1}{P'(m_1)} > \frac{P(m_1)/m_1}{P'(\tilde{m}_1)}, \tag{4}$$

contradicting the equality (3) above. ■

Proposition 1 states the novel form of market failure identified in this paper: under reasonable conditions on the discovery probability function P, the equilibrium direction of innovation is inefficient, as R&D firms over-invest in hot R&D lines. For clarity of exposition, this result is shown here in a simple model. The next section shows how to use this model to build a 'canonical' dynamic framework.

4 Extended Analysis

In this section, we enrich the model of section 3, and build a dynamic framework with canonical assumptions about innovation discovery and R&D competition. We then extend the framework, to incorporate different features of interest, and single out the specific aspects of R&D competition that lead to this paper's novel market inefficiency results.

A Canonical Dynamic Model We begin by extending the model of section 3 to allow for a large number (in fact, a continuum) of R&D lines, with innovation values $z \geq 0$. We say that the innovation value is distributed according to the cumulative distribution function F, which we assume to be twice differentiable. Again, there is a mass $M > 0$ of researchers, who are allocated to the different R&D lines according to a measurable function m. For each innovation of value z, we denote by $m(z)$ the mass of researchers competing for the discovery of that innovation. Hence, the resource constraint $\int_0^\infty m(z)\,dF(z) \leq M$ needs to be satisfied. The probability that an innovation of value z is discovered is denoted by $P(m(z))$, again, assumed increasing in $m(z)$. The value of participating in the R&D race for each innovation of value z is now $U(z; m(z)) = zP(m(z))$. Maintaining the earlier additivity assumption, the welfare associated with allocation m is expressed as $W(m) = \int_0^\infty zP\left(m\left(z\right)\right)dF\left(z\right).$

As in the simpler the model of section 3, letting m and $\tilde{m}$ be the equilibrium and optimal allocation functions, the quantity $\frac{P(m(z))/m(z)}{P'(\tilde{m}(z))}$ needs to be constant over the active R&D lines z, for which $m\left(z\right) > 0$ and $\tilde{m}\left(z\right) > 0$. And, again, when the generalized reverse hazard rate $\Gamma\left(m\right) = mP'\left(m\right)/P(m)$ strictly decreases in m, competing firms overinvest in the hot R&D lines; i.e., formally, there exists a threshold $\bar{z}$ such that $m\left(z\right) < \tilde{m}\left(z\right)$ for $z < \bar{z}$ and $m\left(z\right) > \tilde{m}\left(z\right)$ for $z > \bar{z}$.[34] Further, because the allocation functions m and $\tilde{m}$ cross only once and need to satisfy the same resource constraint, it is also the case that the smallest active R&D line innovation value is higher in equilibrium than in the first best; i.e., that $\tilde{z}_0 = \inf_z\{\tilde{m}\left(z\right) > 0\} \leq z_0 = \inf_z\{m\left(z\right) > 0\}$.

We now introduce explicit dynamics in the model, momentarily assuming that the allocation of researchers is chosen at time zero, and that researchers cannot be redeployed across R&D lines afterwards. Given any R&D allocation function m, we express the expected value for engaging in any R&D line of innovation value z as:

$$U\left(z;m\right) = \int_0^\infty ze^{-rt}p\left(t,m\left(z\right)\right)dt, \tag{5}$$

where r is the discount rate, and $p\left(t,m\left(z\right)\right)$ is the density of the discovery of any innovation of value z at time t, given that a mass $m\left(z\right)$ of researchers investigates on each innovation of value z. We assume that the cumulative distribution function associated with the density $p\left(t,m\left(z\right)\right)$ increases in $m\left(z\right)$ in first-order stochastic sense. The formulation (5) can be subsumed into our earlier extended model, simply by redefining the function P as $P\left(m(z)\right) = \int_0^\infty e^{-rt}p\left(t,m(z)\right)dt,$ for all $m(z) \geq 0$.[35] As a result, we can again express the value of each R&D line of innovation value z as $U\left(z;m(z)\right) = zP\left(m\left(z\right)\right).$

Following the same arguments that led to Proposition 1, we obtain again that R&D firms overinvest in the hot R&D lines, in equilibrium, when $P\left(\hat{m}\right)$ is a concave function, and the generalized reverse hazard rate $\Gamma\left(\hat{m}\right) = \hat{m}P'\left(\hat{m}\right)/P(\hat{m})$ strictly decreases in $\hat{m}$. Most

[34] The proof of this result is entirely analogous to the proof of Proposition 1, and so it is omitted.

[35] The expression $P\left(m\left(z\right)\right)$ is not a probability of discovery, here. Rather it represents the expected discount factor of each innovation of value z. Evidently, as $m\left(z\right)$ increases, the discount factor increases.

importantly, these conditions hold when the density $p(t, \hat{m})$ takes the canonical exponential form $p(t, \hat{m}) = \hat{m}\lambda e^{-\hat{m}\lambda t}$. This expression represents instances in which each innovation's random discovery time is an independent event, and each engaged researcher is equally likely to make the discovery, with time constant hazard rate λ.

With simple manipulations, in fact, we see that in this case:

$$P(\hat{m}) = \int_0^{\infty} e^{-rt}\hat{m}\lambda e^{-\hat{m}\lambda t}dt = \frac{\hat{m}\lambda}{r + \hat{m}\lambda},$$

which is increasing and concave in $\hat{m}$, and that the generalized reverse hazard rate takes the form $\Gamma(\hat{m}) = r/(r + \hat{m}\lambda)$, which strictly decreases in $\hat{m}$. Further, we prove in appendix B that (as long as the resource constraints bind), the equilibrium and optimal allocation functions take m and $\tilde{m}$ the simple forms reported in the following result.

Proposition 2 *Suppose that there is a continuum of R&D lines, whose innovation discoveries are independent events, equally likely among each engaged researcher, with time constant hazard rate λ. Then, the equilibrium and optimal allocation functions are*

$$m(z) = \frac{z - z_0}{\pi}, \quad \text{for all } z \geq z_0 = r\pi/\lambda \tag{6}$$

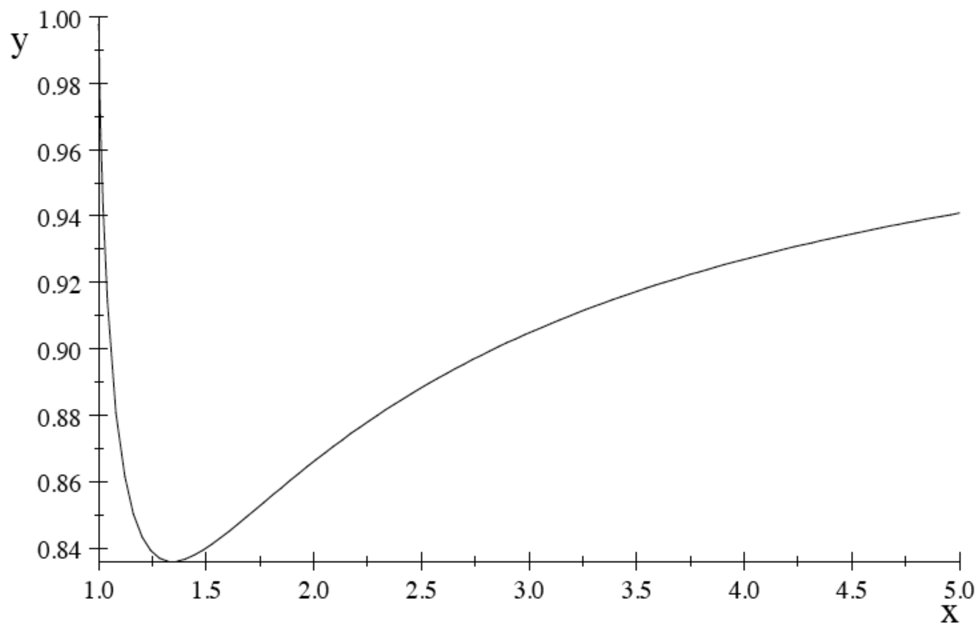
$$\tilde{m}(z) = \frac{r}{\lambda}\left(\sqrt{\frac{z}{\tilde{z}_0}} - 1\right), \quad \text{for } z \geq \tilde{z}_0 = r\mu/\lambda, \tag{7}$$

where π is the equilibrium profit of each R&D line, and μ is the Lagrange multiplier of the resource constraint. In equilibrium, R&D firms overinvest in the hot R&D lines relative to the optimal allocation of researchers: there exists a threshold $\bar{z}$ such that $m(z) < \tilde{m}(z)$ for $z < \bar{z}$ and $m(z) > \tilde{m}(z)$ for $z > \bar{z}$.

Importantly, this result demonstrates the market inefficiency that is the theme of this paper (that competing firms overinvest in hot R&D lines) within a canonical dynamic model directly comparable with the many R&D models since Loury (1979) and Reinganum (1981), that are also built on the assumption of exponential arrival of innovation discoveries.

To get a sense of how meaningful this distortion might be, we perform a simple back-of-the-envelope calculation. Supposing that innovation values are distributed according to

Figure 1: Welfare wedge $W(m)/W(\tilde{m})$ from Proposition 3



a Pareto distribution of parameter $\eta > 1$, so that $F(z) = 1 - z^{-\eta}$ for $z \geq 1$, we derive the following result, proved in appendix B.

Proposition 3 *Consider the canonical dynamic model described in Proposition 2, and suppose that the innovation values z are distributed according to a Pareto distribution of parameter $\eta > 1$. Then the welfare wedge is:*

$$\frac{W(m)}{W(\tilde{m})} = \frac{\eta - 1}{\eta} \left(\frac{2\eta - 1}{\eta - 1} \right)^{1/\eta}. \tag{8}$$

The significance of this result is that it determines a simple expression for welfare loss. Plotting the welfare-wedge expression (8), we see that the welfare loss is negligible for η close to 1, but that it quickly increase as η grows, so that the welfare loss $1 - W(m)/W(\tilde{m})$ reaches its maximum of about 16% for η close to 1.34 to then slowly decrease and disappear asymptotically as $\eta \to \infty$.

In the remainder of the section, we perform variations on the canonical dynamic we developed here. This allows us to single out different features of R&D competition that lead to the market inefficiency finding that is the theme of this paper. By doing this, we complement the general condition on the generalized reverse hazard rate Γ, uncovered in the simpler model of section 3.

Redeployment of Researchers We relax the assumption that the time-0 allocation cannot be changed over time. Suppose that, at any point in time, each researcher can be moved across research lines by paying a switching cost $c \geq 0$. For every innovation of value z and time t, we denote the mass of engaged researchers as $m(t, z)$, and let $z_0(t)$ to be the smallest active R&D line innovation value at time t; i.e., $z_0(t) = \inf_z\{m(t, z) > 0\}$.

Due to the stationarity of the problem, it is never the case that R&D firms choose to move researchers across R&D lines in equilibrium, nor that it is optimal to do so, unless the R&D line in which the researchers are engaged is exhausted as a consequence of innovation discovery. So, we can approach again the problem using standard dynamic programming techniques. We express the equilibrium value $v(z, t)$ of a researcher engaged in a R&D line of innovation value z at t through the Belman equation:

$$rv(z,t) = \lambda m(z,t)\left[\frac{z}{m(z,t)} - c\right] + v_t(z,t), \tag{9}$$

where $v_t(z,t)$ denotes the time-derivative of $v(z,t)$. The flow equilibrium value $rv(z,t)$ includes two terms. The first one is the expected net benefit due to the possibility of innovation discovery. The hazard rate of this event is $\lambda m(z,t)$; if it happens, each researcher gains z with probability $1/m(z,t)$, and pays the cost c to be redeployed to a different R&D line. The second term, $v_t(z,t)$, is the time value change, due to the redeployment of researchers into the considered R&D line from exhausted research lines with discovered innovations.

For any time t, both the equilibrium value $v(z,t)$ and its derivative $v_t(z,t)$ are constant across all active R&D lines of innovation value $z \geq z_0(t)$.[36] Letting $v(t)$ and $v'(t)$ be such

[36]These conditions are akin to value matching and smooth pasting conditions in stopping problems (for example, see Dixit and Pindyck, 1994). Because R&D firms are competitive, and labor is a continuous factor, the equilibrium dissipates all value differences from discovery of different innovations, through congestion and costly redeployment of researchers. This is as akin to the phenomenon of rent dissipation in models of patent races with costly entry.

constants, we obtain the no-arbitrage equilibrium condition:

$$\lambda\left[z-cm\left(z,t\right)\right]=rv(t)-v'(t),\ \text{for all } z\geq z_0(t). \tag{10}$$

Differentiating expression (10) with respect to z, we obtain:

$$m_z\left(z,t\right)=1/c, \tag{11}$$

where $m_z\left(z,t\right)$ is the derivative of $m\left(z,t\right)$ with respect to z. In words, the net expected individual gain for engaging in a marginally more valuable R&D line (this gain equals 1, here, by construction) is exactly offset by the additional individual expected switching cost $c\cdot m_z\left(z,t\right)$; note that, here, $m(z,t)$ is the hazard rate of completion of the R&D race for an innovation of value z and paying the switching cost c, at time t.

Integrating the simple differential equation (11), we obtain the general solution $m\left(z,t\right)=\left[z-z_0(t)\right]/c$, for $z\geq z_0(t)$. When the resource constraint $\int_{z_0(t)}^{\infty}m\left(z,t\right)dG\left(t,z\right)\leq M$ binds, all R&D firms make a non-negative profit by participating to the economy, and the initial condition $z_0(t)$ is pinned down by the equation:

$$cM=c\int_{z_0(t)}^{\infty}m\left(z,t\right)dG(t,z)=\int_{z_0(t)}^{\infty}zdG\left(t,z\right)-z_0(t)[1-G\left(t,z_0(t)\right)], \tag{12}$$

where $G\left(t,z\right)$ is the cumulative distribution function of innovations not discovered yet at time t.

We also note that, because active R&D lines with innovation value $z\geq z_0(t)$ get exhausted over time, more researchers engage in the remaining lines, i.e., $m_t(z,t)>0$ for all $z\geq z_0(t)$; less valuable lines becoming active, i.e., $z_0'(t)<0$, and each active research line becomes less valuable, i.e., $v'(t)<0$. Indeed, the value $v(t)$ decreases over time until the time T such that $v(T)=c$. At that time, R&D firms stop redeploying researchers at the end of the R&D race in which they are engaged. Active research lines have become so congested, that their value is not sufficient to recover the redeployment cost c any longer.[37]

The next Proposition summarizes the above equilibrium analysis.

[37] The characterization of the allocation $m(z,t)$ of researchers on undiscovered R&D lines at any time $t\geq T$ is covered by the earlier analysis of the canonical dynamic model without redeployment of researchers (cf. Proposition 2). In our set up with a continuum of R&D lines distributed according to the twice differentiable function G, arguments invoking 'laws of large number' suggest that the allocation $m(z,t)$ would smoothly converge to the allocation $m(t)$ described in Proposition 2.

Proposition 4 *The equilibrium allocation function m of the canonical dynamic model in which researchers can be moved across R&D lines at cost c is:*

$$m(z,t) = \frac{z - z_0(t)}{c}, \text{ for all } z \geq z_0(t), \tag{13}$$

where the boundary $z_0(t)$ solves equation (12). Researchers are redeployed into different active R&D lines only until the time T such that $z_0(T) = rc/\lambda$, and only if their research line is exhausted due innovation discovery.

We now turn to assess the welfare properties of the market equilibrium allocation m. To do so, we determine the first best allocation function $\tilde{m}$ by solving the problem of a social planner who remunerates individual researchers for allocating across R&D lines optimally over time. In absence of moral hazard, and with complete information, this program achieves the first best (see, for example, Hurwicz, 1960).

Further, as we discussed earlier, this approach to solve for the first best allocation clarifies that the source of market inefficiency, here, are missing property rights and missing markets. Suppose that markets rewarded research up-front, i.e., the *attempt* to achieve marketable innovations, instead of only the *achievement and marketing* of these innovations, through IP rights. Then, by the Coase Theorem, the negative externality exerted on the R&D firms by the individual researchers who engage in hot R&D lines would be internalized. Researchers in less profitable R&D lines would be remunerated above and beyond the expected market prospects of their R&D lines, so as to rebalance remunerations across R&D lines and achieve the first best allocation.[38]

So, suppose that a social planner provides the flow remuneration $u(t)$ to each individual researcher, regardless of the R&D line in which she is engaged.[39] The social planner chooses the optimal allocation function $\tilde{m}$ so as to solve the following program:

$$r\tilde{v}(t,z) = \max_{\hat{m} \in \mathbb{R}} \lambda \hat{m} \left[z - \tilde{v}(t,z) - \hat{m}c \right] - u(t)\hat{m} + \tilde{v}_t(t,z), \text{ for all } z \geq \tilde{z}_0(t), \tag{14}$$

[38] But of course, this is impossible: it is well understood that markets cannot efficiently reward research up-front, because of impediments based on incomplete information, moral hazard, and R&D secrecy.

[39] The remuneration $u(t)$ need to be decreasing over time, because the measure of undiscovered R&D lines decreases over time.

under the constraint that $u(t)$ satisfies the clearing condition in the R&D researchers labor market. The expression $\tilde{v}(t,z)$ denotes the (optimal) social value of engaging researchers in R&D lines of innovation of value z. Innovation discovery has hazard rate $\lambda\tilde{m}(z,t)$, and accrues the value z to the society, but leads to the cost $\tilde{m}(z,t)c$ of redeploying all $\tilde{m}(z,t)$ researchers previously engaged in the R&D of the discovered innovation. The remuneration of researchers, $u(t)\,\tilde{m}(z,t)$, is also made explicit in the planner's program.

The solution of program (14) leads to the first order conditions:

$$\lambda\left[z-\tilde{v}(t,z)-2\tilde{m}(z,t)c\right]=u(t)\,,\ \text{for every } z\geq\tilde{z}_0(t). \tag{15}$$

Equating these first order conditions leads to the differential equation

$$\tilde{m}_z(z,t)=\frac{1-\tilde{v}_z(t,z)}{2c}. \tag{16}$$

Comparing equations (11) and (16), we see that the derivative $\tilde{m}_z$ of the optimal allocation function $\tilde{m}$ is smaller than m_z, the derivative of the equilibrium allocation function m.[40] Because both functions m and $\tilde{m}$ need to satisfy the same resource allocation constraint, we recover once again the market inefficiency result that is the theme of this paper: R&D firms overinvest in the hot research areas.

Further, the comparison of equations (11) and (16) allows us to single out two separate effects that lead to this result. The first one consists in a negative externality on the net benefit for engaging researchers in more profitable R&D lines, which is not internalized in equilibrium. Deploying an additional researchers in a more profitable R&D line makes it marginally more likely that the more profitable innovation is discovered earlier. Hence, in the optimal solution $\tilde{m}$, the net benefit for engaging an additional researcher in a marginally more valuable R&D line is $1-\tilde{v}_z(t,z)$, smaller than the analogous net benefit in equilibrium, which is equal to 1. This difference induces an 'anticipation effect' that pushes towards overinvestment in the hot R&D lines, in equilibrium.

At the same time, the additional social marginal cost for engaging an additional researcher in a marginally more profitable line, $2c\cdot\tilde{m}_z(z,t)$, is twice the private additional

[40] Note that $\tilde{v}_z(t,z)>0$: the social value of researching undiscovered innovations increases in their value z.

expected cost $c \cdot m_z(z,t)$ incurred by the individual researcher. On top of this private cost, in fact, the society suffers also the extra switching cost incurred in expectation by all researchers already engaged in the more profitable R&D line, in case the additional researcher wins the R&D race. This 'switching cost externality' also pushes towards equilibrium overinvestment in the hot R&D lines.[41]

These findings single out important substantive reasons for the market failure identified in this paper, beyond the mathematical condition on the generalized reverse hazard rate Γ presented in Proposition 1. The following result summarizes the analysis of this subsection.

Proposition 5 *In the equilibrium of the canonical dynamic mode, with cost c of moving researchers across R&D lines, firms overinvest in the hot R&D lines at every time t: there exists a threshold function $\bar{z}(t)$ such that $m(z,t) < \tilde{m}(z,t)$ for $z < \bar{z}(t)$ and $m(z) > \tilde{m}(z,t)$ for $z > \bar{z}(t)$.*

In a later part of this section, we modify the canonical dynamic model with redeployment of researchers we developed here. We stipulate that novel profitable R&D lines exogenously arise over time, and replace exhausted R&D lines with discovered innovations, so as to keep the economy in steady state. Before doing this, we devote the next subsection to consider duplicative efforts. We show that duplicative efforts constitute a further source for the market inefficiency we identified in this paper (over-investment in hot R&D lines), beyond and independently of the costly switching of researchers across R&D lines.

Duplicative Efforts As discussed in the introduction, duplicative efforts have long been recognized as a possible source of market inefficiency in R&D competition, leading to the possibility of excessive private investment in R&D. To model duplicative efforts, here, we modify our canonical dynamic model so that the arrival rate of an innovation increases less than linearly in the mass of engaged researchers. In the process of achieving

[41] The result that R&D firms overinvest in hot R&D lines fails to hold only when $c = 0$ (the case of perfectly costless redeployment of researchers). In this case, assuming that the innovation value distribution has bounded support, all researchers will be first engaged in the most valuable R&D lines. When these innovation are discovered, they will all be redeployed to marginally less valuable ones, until also these innovations are discovere, and so on and so forth. This unique equilibrium outcome is also socially optimal.

a patentable innovation, competing firms often need to go through the same intermediate steps, and this occurs independently of every other firms' intermediate results, that are jealously kept secret. Hence, the arrival rate of an innovation often does not double if twice as many firms compete in the same R&D race.

Specifically, maintaining the assumption that innovation arrivals are independent events, we make the following assumptions. For each researcher engaged in a R&D line of innovation value z at time t, the innovation discovery arrival rate $\lambda\left(m(z,t)\right)$ is a twice differentiable and decreasing function of the mass of engaged researchers $m(z,t)$. Further, the aggregate innovation discovery arrival rate $\lambda\left(m(z,t)\right)m(z,t)$ is an increasing and strictly concave function of $m(z,t)$. To isolate the effect of duplicative efforts from the effect of switching costs studied earlier, we here focus on $c=0$.

Obvious modifications in the formulation and analysis of expression (9) imply that, here, the expected innovation value $\lambda(m(z,t))z$ is constant across all active R&D lines with innovation values $z \geq z_0(t)$, in equilibrium. So, differentiating $\lambda(m(z,t))z$ with respect to z, and equating the derivative to zero, we obtain that the equilibrium allocation function m is pinned down by the differential equation

$$m_z(z,t) = -\frac{\lambda(m(z,t))}{\lambda'(m(z,t))m(z,t)} \cdot \frac{1}{z/m(z,t)}. \tag{17}$$

The derivative of the allocation function $m(z,t)$ in z equals the inverse of the innovation value per-researcher $z/m(z,t)$, times the inverse of the elasticity $\epsilon\left(m(z,t)\right) = -m(z,t)\lambda'\left(m(z,t)\right)/\lambda\left(m(z,t)\right)$ of the individual arrival rate $\lambda(m(z,t))$ as a function of the researcher mass value $m(z,t)$.

Turning to the calculation of the optimal allocation function $\tilde{m}$, obvious modifications in the formulation and analysis of expression (14) yield the following first-order conditions:

$$\left[\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)\right]\left[z - \tilde{v}\left(t,z\right)\right] = u(t), \quad \text{for all } z \geq \tilde{z}_0(t). \tag{18}$$

Differentiating this expression with respect to z, and rearranging, we obtain that the optimal allocation function $\tilde{m}$ is determined by the differential equation:

$$\tilde{m}_z(z,t) = -\frac{\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)}{\left[2\lambda'(\hat{m}) + \lambda''(\hat{m})\hat{m}\right]m(z,t)} \cdot \frac{\left[1 - \tilde{v}_z(t,z)\right]z}{z - \tilde{v}(t,z)} \cdot \frac{1}{z/m(z,t)}. \tag{19}$$

This expression is similar than its equilibrium analog, equation (17): the right-hand side equals the inverse of the per-researcher innovation value $z/\tilde{m}(z,t)$ at t, multiplied for (i) the inverse of the elasticity of $\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)$ —the first derivative of the aggregate arrival rate $\lambda(\tilde{m}(z,t))\tilde{m}(z,t)$ of any innovation of value z— as a function of $\tilde{m}(z,t)$, and (ii) the elasticity of the net innovation discovery value $z - \tilde{v}(t,z)$, as a function of z.

As in the previous subsection, the market inefficiency result that R&D firms overinvest in hot research areas is proved in appendix B by showing that the derivative of the optimal allocation function $\tilde{m}$ with respect to the innovation value z is smaller than the derivative of the equilibrium allocation function m, for any time t. Here, however, we need to impose the additional assumption that the elasticity $\epsilon(\hat{m})$ of $\lambda(\hat{m})$ does not decreases in $\hat{m}$.

Proposition 6 *Consider a canonical dynamic model with costless redeployment of researchers across R&D lines, but with duplicative effort within each R&D line: the per-researcher discovery arrival rate $\lambda(m(z,t))$ of each innovation of value z at any time t decreases in the mass $m(z,t)$ of engaged researchers. Suppose that $\lambda(\hat{m})\hat{m}$ is an increasing and concave function of $\hat{m}$, and that the elasticity $\epsilon(\hat{m}) = -\hat{m}\lambda'(\hat{m})/\lambda(\hat{m})$ does not decrease in $\hat{m}$. Then, in equilibrium, R&D firms overinvest in the hot R&D lines: there exists a twice differentiable threshold function $\bar{z}$ such that $m(z,t) < \tilde{m}(z,t)$ for $z < \bar{z}(t)$ and $m(z,t) > \tilde{m}(z,t)$ for $z > \bar{z}(t)$.*

The comparison of equations (17) and (19) leads to identify two separate effects that cause the result stated in Proposition 6. The first one is again a form of the anticipation effect identified earlier. Here, it comes from the elasticity of the net innovation discovery social value $z - \tilde{v}(t,z)$ being smaller than 1, the elasticity of innovation discovery value z. This anticipation effect is reinforced by a 'technological congestion' effect. As we show in appendix B, when the elasticity $\epsilon(\tilde{m}(t,z))$ of the arrival rate $\lambda(\tilde{m}(t,z))$ as a function of $\tilde{m}(t,z)$ weakly increases in $\tilde{m}(t,z)$, it is larger than the inverse of the elasticity of $\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)$, as a function of $\tilde{m}(z,t)$. Intuitively, the duplicative efforts reinforce the negative externalities exerted by R&D firms on their competitors in the same patent races; and these externalities are the source of the result that R&D firms overinvest in the hot R&D lines.

To summarize our findings in this section, within a canonical dynamic framework we have singled out two separate substantive sources for the kind of market failure identified in this paper. Research firms overinvest in the hot R&D lines in equilibrium, because the movement of researchers across R&D lines is costly and/or because of duplicative efforts in R&D races.

Steady State Economy We now return to the canonical dynamic model with costly redeployment of researchers across R&D lines, and without duplicative efforts within R&D races. We introduce the possibility of successive innovation vintages, and suppose that novel profitable R&D lines arise over time and replace exhausted R&D lines that already led to the discovery of profitable innovations. We focus on R&D line replacement that keeps the economy in steady state. Letting $m(z)$ and $\tilde{m}(z)$ be the stationary market equilibrium and optimal allocation functions, respectively, the flow arrival rate of R&D lines is denoted by α, and the cumulative distribution function of novel R&D line innovation values by F, with associated density f.

The analysis begins by noticing that both the equilibrium and optimal allocation functions, $m(z)$ and $\tilde{m}(z)$, and the associated probability densities g and $\tilde{g}$ of undiscovered innovation values, satisfy the stationarity conditions:

$$\lambda m(z) g(z) = \alpha f(z), \text{ for all } z \geq z_0, \tag{20}$$

$$\lambda \tilde{m}(z) \tilde{g}(z) = \alpha f(z), \text{ for all } z \geq \tilde{z}_0. \tag{21}$$

For any innovation value z with active R& lines, the total mass of researchers allocated in the steady state equilibrium and optimal allocations, respectively $m(z) g(z)$ and $\tilde{m}(z) \tilde{g}(z)$, are both equal to $(\alpha/\lambda) f(z)$, the net inflow of R&D lines of innovation value z. Of course, this does not mean that also the mass of researchers engaged in each R&D line is the same: it need not be that $m(z) = \tilde{m}(z)$ for any active R&D line of innovation value z. But it must be that the support of $m(z) g(z)$ and $\tilde{m}(z) \tilde{g}(z)$ coincide in steady state, so that the smallest active R&D line innovation values z_0 and $\tilde{z}_0$ also coincide.

Turning to calculating the equilibrium, obvious modifications of the analysis presented earlier in the section imply that, again, the expression $\lambda [z - m(z) c]$ is constant for all $z \geq z_0$, that the equilibrium satisfies the differential equation $m_z(z) = 1/c$, and that the equilibrium expression is $m(z) = (z - z_0)/c$, for all $z \geq z_0$. When the resource feasibility constraint $\int_{z_0}^{+\infty} m(z) g(z) dz \leq M$ binds, all R&D firms make a non-negative profit by participating to the economy, and the threshold z_0 is pinned down by plugging the stationarity condition (20) into the binding resource constraint, so as to obtain the equation

$$\alpha (1 - F(z_0)) = \lambda M.$$

As a result, we see that z_0 is determined independently of the allocation function m, here, and we prove in appendix B that the stationary equilibrium value $\bar{v}$ of engaging researchers in every R&D line of innovation value $z \geq z_0$ is $\bar{v} = (\lambda/r) z_0$. [42] This result is analogous to rent dissipation in the original literature on patent races. While in that literature an elastic supply of researchers implied all rents where drawn to the opportunity cost, in our case with inelastic supply all rents are equalized and driven to z_0, the least valuable innovation. The differential value of better innovation is dissipated through switching costs. Finally, by returning to the stationarity condition (20), we calculate the density of G:

$$g(z) = (\alpha/\lambda) \frac{f(z) c}{z - z_0}.$$

With obvious modifications of the Belman equations (14), the social planner problem takes the following form, here:

$$r\tilde{v}(z) = \max_{\hat{m} \in \mathbb{R}} \lambda \hat{m} [z - \tilde{v}(z) - \hat{m}c] - u\hat{m}, \quad \text{for all } z \geq z_0, \tag{22}$$

under the constraint that u satisfies the clearing condition in the R&D researchers labor market. The associated first-order conditions are:

$$\lambda [z - \tilde{v}(z) - 2\tilde{m}(z)c] = u, \quad \text{for every } z \geq z_0. \tag{23}$$

[42] If the resource constraint is satisfied with a strict inequality, $\int_{z_0}^{\infty} m(z) dG(z) < M$, then the economy cannot support entry by all firms, the participation constraint $\bar{v} \geq c$ binds and pins down z_0 through the equality $c = \bar{v} = (\lambda/r) z_0$.

With simple manipulations presented in appendix B, we thus obtain:

$$\lambda\left[z - c\frac{\lambda\tilde{m}(z)^2}{r} - 2c\cdot\tilde{m}(z)\right] = u, \quad \text{for every } z \geq z_0. \tag{24}$$

These equations are analogous to the first-order conditions (15) in the canonical dynamic model without R&D line replacement we solved earlier. The only difference is that the term $\tilde{v}(z)$ takes the constant form $c\cdot\lambda\tilde{m}(z)^2/r$, here, which is the discounted cost of all future redeployment of the mass $\tilde{m}(z)$ of researchers engaged in the considered R&D line —the term $c\cdot\lambda\tilde{m}(z)/r$ is the individual discounted cost. So, we can identify as $c\cdot\lambda\tilde{m}(z)^2/r + c\cdot\tilde{m}(z)$, the 'switching cost externality' that an additional researcher imposes on the $\tilde{m}(z)$ researchers engaged in the R&D line.[43]

Applying the implicit function theorem to expression (24), we obtain the differential equation.

$$\tilde{m}'(z) = \frac{1}{2c}\left(\frac{1}{\lambda\tilde{m}(z)/r+1}\right). \tag{25}$$

Further, the optimal allocation function $\tilde{m}$ can be solved explicitly from expression (24), here. We show in appendix B that is takes the simple closed form:

$$\tilde{m}(z) = \frac{r}{\lambda}\left(\sqrt{\lambda\frac{z-\tilde{z}_0}{rc}+1} - 1\right), \text{ for all } z \geq z_0. \tag{26}$$

Evidently, the derivative of $\tilde{m}$, reported in expression (25), is strictly smaller than $1/c$, the derivative of $m(z)$ reported earlier. Once again, we have reached the conclusion that R&D firms overinvest in the hot R&D lines in equilibrium. We highlight this result in the following proposition.

Proposition 7 *Consider the canonical dynamic model with costly redeployment of researchers, and replacement of exhausted R&D lines that keeps the distribution of undiscovered innovation values stationary. In equilibrium, R&D firms overinvest in the hot R&D lines: there exists a threshold $\bar{z}$ such that $m(z) < \tilde{m}(z)$ for $z < \bar{z}$, and $m(z) > \tilde{m}(z)$ for $z > \bar{z}$.*

[43] The net benefit of an additional researcher in the R&D line equals this researcher's discovery hazard rate λ, multiplied by the innovation value z, minus the current and future discounted switching costs $c\tilde{m}(c) + c\lambda\tilde{m}(z)^2/r$ borne by the other $\tilde{m}(z)$ researchers, minus the cost of $\tilde{m}(z)c$ of redeploying this marginal researcher. The latter, grouped with z, gives the expression $\lambda[z - \tilde{m}(z)]c$ which is the private marginal net benefit of researchers in the R&D line, as reported earlier.

Because the support of the allocation functions m and $\tilde{m}$ coincide, it is also the case that the thresholds z_0 and $\bar{z}$ coincide, so that the functions m and $\tilde{m}$ cross at z_0, the smallest innovation value for which the R&D lines are active. Together with the fact that $m' > \tilde{m}'$, this implies that $m > \tilde{m}$, that $g < \tilde{g}$ and that $g' < \tilde{g}'$. And to make up for the fact that $m(z_0) = \tilde{m}(z_0) = 0$, it must also be that $\lim_{z \to z_0^+} g(z) = \lim_{z \to z_0^+} \tilde{g}\,(z) = +\infty$. In words, the density of the R&D lines with undiscovered innovations is very large for small innovation values, very few researchers are engaged on these R&D lines, and hence innovation discoveries arrive with a very low rate. As the innovation value grows larger, the density of R&D lines with undiscovered innovations decreases. The rate of decrease is larger for the competitive equilibrium than for the optimal allocation function. So, the market suboptimally exhausts too many high value R&D lines too early, and leaves too few for future discovery.

We conclude the analysis of the stationary economy, by showing that the equilibrium and optimal welfare functions take the following simple closed forms.

Proposition 8 *Consider the canonical dynamic model with costly redeployment of researchers and stationary distribution of R&D lines with undiscovered innovations. The aggregate equilibrium welfare is:*[44]

$$W(m) = (\lambda/r) z_0 M \tag{27}$$

The aggregate welfare associated with the optimal allocation $\tilde{m}$ is:

$$W(\tilde{m}) = \frac{\alpha}{r} \int_{z_0}^{\infty} z f\,(z)\, dz + cM - \frac{\alpha}{\lambda} \int_{z_0}^{\infty} \sqrt{\frac{c\lambda}{r}\,(z - z_0) + c^2} \cdot f\,(z)\, dz. \tag{28}$$

These closed-form expressions make welfare assessments simple and precise.

When the switching costs are small, the optimal welfare expression simplifies further:

$$\begin{aligned} \lim_{c \to 0^+} W(\tilde{m}) &= (\alpha/r) \int_{z_0}^{\infty} z f\,(z)\, dz = (\alpha/r) E\,(z | z \geq z_0)\,[1 - F(z_0)] \\ &= (\lambda/r) E\,(z | z \geq z_0)\, M. \end{aligned}$$

[44] It may appear surprising that the steady state equilibrium welfare $W(m)$ is independent of c. This follows from the fact that the equilibrium deployment cost $m\,(z)\,c$ equals $z - z_0$ independently of c for any innovation value $z \geq z_0$.

Again, this result makes transparent the comparison between rent dissipation in the competitive equilibrium that is not present in the planner's solution. Thus, for small switching cost c, the welfare ratio $W(m)/W(\tilde{m})$ takes the form:

$$\lim_{c \to 0^+} \frac{W(m)}{W(\tilde{m})} = \frac{z_0}{E\left(z|z \geq z_0\right)}.$$

In words, the welfare ratio converges to the smallest active R&D line innovation value z_0, divided by the expected active R&D line innovation value. It is intuitive that this quantity can be significantly small, for standard cumulative distributions F.[45]

5 Conclusion

Research on the efficiency of innovation markets is usually concerned on whether the level of R&D firm investment is socially optimal. This paper has asked a distinct, important question: Does R&D go in the right direction? In a simple, general model, we have demonstrated that R&D competition pushes firms to disproportionately engage in the hot research lines, characterized by higher expected rates of return. The identification of this form of market failure is a novel result, as we explained in details in the introduction and literature review.

After demonstrating this result within a simple framework, we have embedded our analysis in a canonical dynamic framework that can be directly compared with extant R&D race models, and that we modified and extended in different directions. This framework is populated with a continuum of innovation R&D lines, whose discovery is an independent random event, equally likely across all engaged researchers, and with time constant hazard rate. We considered different variations in which researchers may or may not be moved across R&D lines costly, in which the innovation discovery hazard rate grows with the number of engaged researchers proportionally, or less than proportionally because of duplicative

[45]We performed a back-of-the-envelope calculation of the welfare ratio $W(m)/W(\tilde{m})$, under the assumption that the distribution F is lognormal with mean equal to 7 and standard deviation equal to 1.5, consistently with the estimates provided by Schankerman (1998). With cost $c = 1$ million, the welfare ratio $W(m)/W(\tilde{m})$ is approximately 0.28. As the cost c vanishes, the ratio $W(m)/W(\tilde{m})$ converges to 0.17 approximately.

R&D effort, and in which R&D lines that are exhausted due to innovation discovery may or may not be replaced by successive innovation vintages.

We have recovered our market inefficiency result that competing R&D firms overinvest in the hot research areas in all cases, with the exception of the knife-hedge case without duplicative R&D effort, without R&D line replacement, and in which researchers can be switched across R&D line costless. Importantly, this analysis allowed us to clarify which specific features of R&D competition lead to the market failure that is the theme of this paper. Finally, we have shown that the equilibrium and optimal welfare expressions take simple closed forms in 'mature' R&D industries, where novel innovation opportunities replace previous innovation vintages so as to keep the economy in steady state.

The implications of our study in terms of policy are transparent. Because of our prediction that competing R&D firms overinvest in the hot research lines in equilibrium, this paper advocates for intervention that rebalances remuneration across R&D lines, so as to subsidize R&D lines with less profitable or less feasible innovations. Importantly, our case for R&D subsidization is based on a framework without any market frictions. We argue that even frictionless markets and competition forces cannot solve the form of market failure identified here, because of an unavoidable missing market institutional constraint. Efficiency could be achieved without external intervention, here, only if markets were capable to fully remunerate R&D effort up-front, instead of just rewarding innovation discovery. But of course, this is not what profit motivated entrepreneurs would do, because of several well-understood impediments caused by incomplete information, moral hazard, and R&D secrecy.

Details of the existing forms and mechanisms of non-market R&D funding have been discussed in the literature review (section 2). The main sources of funding are research grants and fiscal incentives, in the form of subsidies or tax breaks. Prizes, procurements and the funding of academia also serve to subsidize R&D, but they do not seem plausibly effective in alleviating the form of market inefficiency identified here (overinvestment in the hot R&D areas). It is even possible that prizes, procurement and career concerns in academia exacerbate the market inefficiency singled out in this paper. Plausibly, they may bias incentives of individual researchers so that they disproportionately compete on a small set of high-profile breakthroughs, instead of spreading their efforts more evenly across valuable innovations.[46]

Returning to comparing grants and fiscal incentives, the extant literature identifies as the main advantage of fiscal incentives, the fact that they leave the choice of the direction of R&D to the informed parties: the competing R&D firms. Unfortunately, this is exactly the fundamental source of the market inefficiency that we have identified in this paper. It is therefore possible that fiscal incentives are ineffective for the purposes advocated here, and that direct State intervention through grants and procurement would be a preferable mechanism. The verification of this concluding conjecture will require extensive work, both in terms of formal modelling and data-based quantitative assessments, and is beyond the boundaries of this paper.

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Appendix A: Omitted Extensions

General Utilities, Externalities and Technological Spillovers Here, we generalize Proposition 1 to any specification of the innovations' values. We normalize $u(0,0)=0$, and let $z_1 = u(1,0)$, $z_2 = u(0,1)$, and $\bar{z} = u(1,1)$ be the innovations' values, where the index 1 stands for the discovery of innovation j and the index 0 stands for non discovery. Now, the social planner problem's is to find $\tilde{m}_1$ and $\tilde{m}_2$ so as to maximize:

$$\begin{aligned} W(\tilde{m}_1, \tilde{m}_2) &= P(\tilde{m}_1)[1-P(\tilde{m}_2)]z_1 + [1-P(\tilde{m}_1)]P(\tilde{m}_2)z_2 + P(\tilde{m}_1)P(\tilde{m}_2)\bar{z} \\ \text{s.t. } \tilde{m}_2 + \tilde{m}_1 &= M. \end{aligned}$$

Equating the first-order conditions, we obtain:

$$\begin{aligned} &\frac{\partial}{\partial \tilde{m}_1} W(\tilde{m}_2, \tilde{m}_1) = P'(\tilde{m}_1)[1-P(\tilde{m}_2)]z_1 - P'(\tilde{m}_1)P(\tilde{m}_2)z_2 + P'(\tilde{m}_1)P(\tilde{m}_2)\bar{z} \\ &= P'(\tilde{m}_1)\{[1-P(\tilde{m}_2)]z_1 + P(\tilde{m}_2)[\bar{z}-z_2]\}. \\ &= P'(\tilde{m}_2)\{[1-P(\tilde{m}_1)]z_2 + P(\tilde{m}_1)[\bar{z}-z_1]\} = \frac{\partial}{\partial \tilde{m}_2} W(\tilde{m}_2, \tilde{m}_1). \end{aligned} \tag{29}$$

The equilibrium condition, instead, is simply:

$$\frac{P(m_2)}{m_2}\{[1-P(m_1)]z_2 + P(m_1)p_2\} = \frac{P(m_1)}{m_1}\{[1-P(m_2)]z_1 + P(m_2)p_1\}, \tag{30}$$

where p_1 and p_2 are the prices of innovations 1 and 2 if both innovations are discovered, in which case, almost surely, the two innovations are discovered by different researchers.

In determining the values of p_1 and p_2 we distinguish two cases. In the first one, the innovations are weak substitutes, so that $\bar{z} \le z_1+z_2$. Here, we set $p_1 = \bar{z}-z_2$ and $p_2 = \bar{z}-z_1$, as this is the unique outcome of competition in which each innovator fully appropriates of the marginal product of its innovation. Indeed, the core of this economy is made of all positive prices (p_1, p_2) such that $p_1 + p_2 \le \bar{z}$ and $p_1 \le \bar{z} - z_2$ and $p_2 \le \bar{z} - z_1$. Hence, the prices $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$ are the best outcome for the innovators in the core, because $p_2 + p_1 = \bar{z} - z_2 + \bar{z} - z_1 \le \bar{z}$, as is implied by the condition $\bar{z} \le z_1 + z_2$. When $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$, dividing the equilibrium condition (30) by the social planner's solution condition (29), we obtain equality (3), again. Hence, the proof of Proposition 1 implies that, again, too many researchers are engaged in the hot R&D line 2, whenever $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m.

In the second case, innovations are strict complements, so that $\bar{z} > z_1 + z_2$. Here, the prices $p_1 = \bar{z} - z_2$ and $p_2 = \bar{z} - z_1$ are not in the core of the economy, because $p_1 + p_2 = \bar{z} - z_2 + \bar{z} - z_1 > \bar{z}$. So, we suppose that the two innovators split the joint innovation profits according to the Nash bargaining solution. The feasible bargaining set corresponds to the core: the set of all positive prices (p_1, p_2) such that $p_1 + p_2 \le \bar{z}$ and $p_1 \le \bar{z} - z_2$ and $p_2 \le \bar{z} - z_1$. The two innovators threat points are z_1 and z_2, because each innovator can at least guarantee to sell its good at its value —recall that $\bar{z} > z_1 + z_2$, so that the consumer is willing to pay the total price $z_1 + z_2$ to buy both innovations. Hence the Nash bargaining solution (p_1, p_2) is such that $p_1 - z_1 = p_2 - z_2$ and $p_1 + p_2 = \bar{z}$. Solving out these two equations, we obtain: $p_1 = \frac{1}{2}[\bar{z} + z_1 - z_2]$, $p_2 = \frac{1}{2}[\bar{z} + z_2 - z_1]$.

So, the equilibrium conditions are:

$$\begin{aligned}&\frac{P(m_2)}{m_2}\left\{[1-P(m_1)]z_2+\frac{1}{2}P(m_1)[\bar{z}+z_2-z_1]\right\}\\ =\;&\frac{P(m_1)}{m_1}\left\{[1-P(m_2)]z_1+\frac{1}{2}P(m_2)[\bar{z}-z_2+z_1]\right\}.\end{aligned} \tag{31}$$

Comparing these conditions with the optimality conditions (29), we conclude below that again, too many researchers are engaged in the hot R&D line 2, whenever $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m.[47]

Proposition A.1. *Suppose that the success probability $P(m)$ is increasing and weakly concave, and that $P(m)/m$ weakly decreases in m. For any specification of the innovations' values, if the generalized reverse hazard rate $\Gamma(m) = mP'(m)/P(m)$ strictly decreases in m, R&D firms overinvest in the hot R&D line 2 in equilibrium; i.e., $m_2 > \tilde{m}_2 > \tilde{m}_1 > m_1$.*

Proof. The proof for the case in which $\bar{z} \le z_1 + z_2$ is entirely analogous to the proof of Proposition 1. Considering the case in which $\bar{z} > z_1 + z_2$, proceeding by contradiction, suppose $m_2 \le \tilde{m}_2$, so that $P(\tilde{m}_1) \le P(m_1) < P(m_2) \le P(\tilde{m}_2)$. We first note that this implies:

$$\frac{[1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1]}{[1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2]} < \frac{2[1-P(m_1)]z_2+P(m_1)[\bar{z}-z_1+z_2]}{2[1-P(m_2)]z_1+P(m_2)[\bar{z}-z_2+z_1]},$$

because

$$\begin{aligned}&([1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1])(2[1-P(m_2)]z_1+P(m_2)[\bar{z}-z_2+z_1])\\ &-(2[1-P(m_1)]z_2+P(m_1)[\bar{z}-z_1+z_2])([1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2])\\ =\;&(\bar{z}-z_1-z_2)\{[2P(\tilde{m}_1)-P(\tilde{m}_1)P(m_2)+P(\tilde{m}_2)P(m_1)-P(m_1)]z_1\\ &-[2P(\tilde{m}_2)-P(\tilde{m}_2)P(m_1)+P(\tilde{m}_1)P(m_2)-P(m_2)]z_2\\ &+[P(\tilde{m}_1)p_2-P(\tilde{m}_2)P(m_1)]\bar{z}\}\\ \propto\;&P(\tilde{m}_1)z_1+(P(\tilde{m}_1)[1-P(m_2)]-P(m_1)[1-P(\tilde{m}_2)])z_1\\ &-P(\tilde{m}_2)z_2-(P(\tilde{m}_2)[1-P(m_1)]-P(m_2)[1-P(\tilde{m}_1)])z_2\\ &+[P(\tilde{m}_1)P(m_2)-P(\tilde{m}_2)P(m_1)]\bar{z}\\ <\;&P(\tilde{m}_1)z_1-P(\tilde{m}_2)z_2<0.\end{aligned}$$

This result, together with the chain of inequalities (4), contradicts the equality

$$\frac{P(m_2)/m_2}{P(m_1)/m_1}\frac{[1-P(m_1)]z_2+\frac{1}{2}P(m_1)[\bar{z}+z_2-z_1]}{[1-P(m_2)]z_1+\frac{1}{2}P(m_2)[\bar{z}-z_2+z_1]} = \frac{P'(\tilde{m}_2)}{P'(\tilde{m}_1)}\frac{[1-P(\tilde{m}_1)]z_2+P(\tilde{m}_1)[\bar{z}-z_1]}{[1-P(\tilde{m}_2)]z_1+P(\tilde{m}_2)[\bar{z}-z_2]},$$

which is implied by the optimal and the equilibrium conditions (29) and (31). ■

[47] This result holds beyond the case of Nash bargaining, here, as long as the share of profits of the holder of the hot innovation patent is not too small.

Heterogenous Arrival Rates Here, we allow for the possibility that the arrival rates of innovations differ across R&D lines. Letting $\lambda(z)$ be the discovery arrival rate of innovations with value z, we see that, now, $P(z, m(z)) = m(z)\lambda(z)/[r + m(z)\lambda(z)]$. The same arguments that lead to Proposition 2 imply that again, R&D firms overinvest in the hot, most attractive, research lines, in equilibrium. Here, however, the attractiveness of an R&D line is not determined by its innovation value z alone, but by the expected flow value $z\lambda(z)$ of engaging in the R&D line. So, we can reformulate and extend Proposition 2 as follows.

Proposition A.2. *Consider the canonical dynamic model in which the discovery arrival rate of innovations with value z is time-constant and equal to $\lambda(z)$. In equilibrium, firms overinvest in the R&D lines with the highest expected flow value $z\lambda(z)$: there exists a threshold ζ such that $m(z) < \tilde{m}(z)$ for $z\lambda(z) < \zeta$ and $m(z) > \tilde{m}(z)$ for $z\lambda(z) > \zeta$.*

Proof. Here, the equilibrium arbitrage conditions require that the expression $\frac{P(z,m(z))}{m(z)} = \frac{z\lambda(z)}{r+m(z)\lambda(z)}$ is constant in z. Likewise, equating the first-order conditions to find the optimal allocation $\tilde{m}$ implies that the expression $P_m(z, \tilde{m}(z))\, z = \frac{rz\lambda(z)}{[r+\tilde{m}(z)\lambda(z)]^2}$ is constant in z.

Take any two values of z, z_1 and z_2 and say without loss of generality that $z_2\lambda(z_2) > z_1\lambda(z_1)$. Because the function $\frac{r}{(r+\tilde{m}\lambda)^2}$ decreases in $\tilde{m}\lambda$, it follows that $\tilde{m}_2\lambda_2 > \tilde{m}_1\lambda_1$, where we write $\tilde{m}_j$ instead of $\tilde{m}(z_j)$ and λ_j instead of $\lambda(z_j)$ for brevity. Dividing the no arbitrage condition

$$\frac{P_1(m_1)}{m_1} z_1 = z_1\lambda_1 \frac{1}{r + m_1\lambda_1} = z_2\lambda_2 \frac{1}{r + m_2\lambda_2} = \frac{P_2(m_2)}{m_2} z_2,$$

where we write m_j instead of $m(z_j)$, by the social planner's solution condition

$$P_1'(\tilde{m}_1)\, z_1 = z_1\lambda_1 \frac{r}{(r + \tilde{m}_1\lambda_1)^2} = z_2\lambda_2 \frac{r}{(r + \tilde{m}_2\lambda_2)^2} = P_2'(\tilde{m}_2)\, z_2,$$

we obtain:

$$\frac{(r + \tilde{m}_1\lambda_1)^2}{r(r + m_1\lambda_1)} = \frac{(r + \tilde{m}_2\lambda_2)^2}{r(r + m_2\lambda_2)}.$$

Now suppose, by contradiction, that $m_2 \leq \tilde{m}_2$ and that $m_1 \geq \tilde{m}_1$. Then, we obtain the contradiction:

$$\frac{(r + \tilde{m}_1\lambda_1)^2}{r(r + m_1\lambda_1)} \leq \frac{(r + \tilde{m}_1\lambda_1)^2}{r(r + \tilde{m}_1\lambda_1)} < \frac{(r + \tilde{m}_2\lambda_2)^2}{r(r + \tilde{m}_2\lambda_2)} \leq \frac{(r + \tilde{m}_2\lambda_2)^2}{r(r + m_2\lambda_2)},$$

using the fact that the function $\frac{(r+\tilde{m}\lambda)^2}{r(r+\tilde{m}\lambda)}$ increases in $\tilde{m}\lambda$. ■

This result provides a useful generalization of our finding that competing firms overinvest in hot R&D lines. In most applications, R&D lines differ both in terms of the expected rate of returns and the expected feasibility of innovations. Because of the canonical nature of exponential arrivals, this generalized result can be easily taken to industry datasets.

Proposition A.2 can be further generalized to broader classes of arrival densities $p(t, z, m(z))$, beyond the canonical exponential class in which $p(t, z, m(z)) = m(z)\lambda(z)e^{-m(z)\lambda(z)t}$, whenever an appropriate parametrization is suitable.

Appendix B: Omitted Proofs

Proof of Proposition 2. For all innovation values z with active R&D lines, the equilibrium no-arbitrage conditions are

$$z\frac{P(m)}{m} = z\frac{\lambda}{r+m\lambda} = \pi.$$

Solving out, we obtain

$$m(z) = \max\{0, z/\pi - r/\lambda\} = r/\lambda \max\{0, z/z_0 - 1\},$$

where $z_0 = r\pi/\lambda$ is the smallest-value R&D line z such that $m(z) > 0$, thus obtaining expression (6).

A social planner chooses $\tilde{m}(\cdot)$ to maximize the social welfare:

$$W(\tilde{m}(\cdot)) = \int_0^\infty z\frac{\tilde{m}(z)\lambda}{r+\tilde{m}(z)\lambda} f(z)\,dz \quad \text{s.t.} \quad \int_0^\infty \tilde{m}(z)\,f(z)dz = M.$$

Hence, the Euler conditions are that, for all z,

$$z\frac{r\lambda}{(r+\tilde{m}(z))\,\lambda)^2} = \mu,$$

where μ is the Lagrange multiplier of the resource constraint. Solving out, we get

$$\tilde{m}(z) = \max\left\{0, \sqrt{(z/\mu)\,(r/\lambda)} - r/\lambda\right\} = \max\left\{0, \sqrt{z/\tilde{z}_0} - 1\right\} r/\lambda,$$

where $\tilde{z}_0 = r\mu/\lambda$ is the smallest-value R&D line z such that $\tilde{m}(z) > 0$, thus obtaining expression (7).

The proof that there exists a threshold $\bar{z}$ such that $m(z) < \tilde{m}(z)$ for $z < \bar{z}$ and $m(z) > \tilde{m}(z)$ for $z > \bar{z}$ is analogous to the proof of Proposition 1, once realized that the generalized inverse hazard rate $\Gamma(\hat{m}) = r/(r+\hat{m}\lambda)$ strictly decreases in $\hat{m}$. ■

Proof of Proposition 3. Returning to expression (6) for the equilibrium allocation m, we note that, here, z_0 is pinned down by the resource constraint:

$$M = \int_0^\infty m(z)f(z)dz = \frac{r}{\lambda}\int_{z_0}^\infty \left(\frac{z}{z_0} - 1\right)\frac{1}{z^{\eta+1}}\eta dz = \frac{r}{\lambda}\frac{z_0}{\eta - 1}.$$

Hence, the equilibrium welfare is simply:

$$W(m) = M\pi = \frac{z_0^{1-\eta}}{\eta - 1},$$

as each researcher earns the value π, and there is a continuum of mass M of researchers.

Returning to expression (7) for the optimal allocation $\tilde{m}$, we see that $\tilde{z}_0$ is pinned down by the resource constraint

$$M = \int_1^\infty \tilde{m}(z)\,dF(z) = \frac{r}{\lambda}\int_{\tilde{z}_0}^\infty \left(\sqrt{\frac{z}{\tilde{z}_0}} - 1\right)\frac{1}{z^{\eta+1}}\eta dz = \frac{r}{\lambda}\frac{\tilde{z}_0^{-\eta}}{2\eta - 1}.$$

Because the expected social value of employing $\tilde{m}(z)$ researchers in any R&D line of value z is

$$\begin{aligned} zP(\tilde{m}(z)) &= z\frac{\tilde{m}(z)\lambda}{r+\tilde{m}(z)\lambda} = z\frac{\frac{r}{\lambda}\left(\sqrt{\frac{z}{\tilde{z}_0}}-1\right)\lambda}{r+\frac{r}{\lambda}\left(\sqrt{\frac{z}{\tilde{z}_0}}-1\right)\lambda} \\ &= z-\sqrt{z\tilde{z}_0}, \end{aligned}$$

integrating over z, we obtain that the optimal welfare is:

$$\begin{aligned} W(\tilde{m}) &= \int_{\tilde{z}_0}^{\infty} zP(\tilde{m}(z))dF(z) = \int_{\tilde{z}_0}^{\infty}\left(z-\sqrt{z\tilde{z}_0}\right)\frac{1}{z^{\eta+1}}\eta dz \\ &= \eta\left[\frac{\tilde{z}_0^{1-\eta}}{(\eta-1)(2\eta-1)}\right]. \end{aligned}$$

Dividing $W(m)$ by $W(\tilde{m})$, we obtain expression (8). ■

Proof of Proposition 4. The equilibrium analysis is presented in the main body of the paper. We now turn to prove the comparative statics results. By the implicit function theorem, totally differentiating equation (12), we obtain:

$$\begin{aligned} z_0'(t) &= -\frac{D_t\left(\int_{z_0}^{\infty} zdG(t,z)-z_0[1-G(t,z_0)]\right)}{D_{z_0}\left(\int_{z_0}^{\infty} zdG(t,z)-z_0[1-G(t,z_0)]\right)} \\ &= \frac{\int_{z_0}^{\infty} zdG_t(t,z)+z_0G_t(t,z_0)}{1-G(t,z_0)}. \end{aligned} \tag{33}$$

Now, we express $G(t,z)$, the cumulative distribution function of undiscovered innovation values yet at time t, as a function of the non normalized cumulative distribution function of the innovation not discovered yet at time t, which we denote by $H(t,z)$, so that $G(t,z) = \int_{-\infty}^{z} dH(t,x) / \int_{-\infty}^{\infty} dH(t,x)$. For clarity, we smooth H and consider the (Radon–Nikodym) derivative h, rewriting $G(t,z)$ as $G(t,z) = \int_{-\infty}^{z} h(t,x)\,dx / \int_{-\infty}^{\infty} h(t,x)\,dx$. Because the arrival rate of each innovation x is $\lambda m(z,x)$, we have that, for small $\Delta > 0$, $\frac{h(t+\Delta,x)-h(t,x)}{\Delta} \approx -\lambda m(x,t)\,h(t,x)$, or $h(t+\Delta,x) \approx h(t,x)[1-\lambda m(x,t)\Delta]$.

Now, we can express the cumulative distribution function $G_t(t,z)$ as:

$$G_t(t,z) = \lim_{\Delta\to 0}\left(\frac{1}{\Delta}\left(\frac{\int_{-\infty}^{z}[1-\lambda m(x,t)\Delta]h(t,x)\,dx}{\int_{-\infty}^{\infty}[1-\lambda m(x,t)\Delta]h(t,x)dx} - \frac{\int_{-\infty}^{z} h(t,x)\,dx}{\int_{-\infty}^{\infty} h(t,x)\,dx}\right)\right),$$

and obtain, by De L'Hopital rule:

$$
\begin{aligned}
G_t\left(t,z\right) &= \lim_{\Delta\to 0}\frac{d}{d\Delta}\left(\frac{\int_{-\infty}^{z}[1-\lambda m\left(x,t\right)\Delta]h\left(t,x\right)dx}{\int_{-\infty}^{\infty}[1-\lambda m\left(x,t\right)\Delta]h(t,x)dx}-\frac{\int_{-\infty}^{z}h\left(t,x\right)dx}{\int_{-\infty}^{\infty}h\left(t,x\right)dx}\right)\\
&= \lim_{\Delta\to 0}\left(\frac{\left[\begin{array}{c}\int_{-\infty}^{z}[-\lambda m\left(x,t\right)]h\left(t,x\right)dx\int_{-\infty}^{\infty}[1-\lambda m\left(x,t\right)\Delta]h(t,x)dx\\ -\int_{-\infty}^{\infty}[1-\lambda m\left(x,t\right)]h(t,x)dx\int_{-\infty}^{z}[1-\lambda m\left(x,t\right)\Delta]h\left(t,x\right)dx\end{array}\right]}{\left(\int_{-\infty}^{\infty}[1-\lambda m\left(x,t\right)\Delta]h(t,x)dx\right)^2}\right)\\
&= \lambda\frac{\int_{-\infty}^{\infty}m\left(x,t\right)h(t,x)dx\int_{-\infty}^{z}h\left(t,x\right)dx-\int_{-\infty}^{z}m\left(x,t\right)h\left(t,x\right)dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}.
\end{aligned}
$$

Substituting in the above expression the formula $m\left(x,t\right)=\max\{[x-z_0(t)]\,/c,0\}$, we obtain that for $z\geq z_0(t)$,

$$
\begin{aligned}
G_t\left(t,z\right) &= \frac{\lambda}{c}\frac{\int_{-\infty}^{z}h\left(t,x\right)dx\int_{-\infty}^{\infty}\left[x-z_0(t)\right]h(t,x)dx-\int_{-\infty}^{z}\left[x-z_0(t)\right]h\left(t,x\right)dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}\\
&= \frac{\lambda}{c}\frac{\int_{-\infty}^{z}h\left(t,x\right)dx\int_{-\infty}^{\infty}xh(t,x)dx-\int_{-\infty}^{z}xh\left(t,x\right)dx\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2},
\end{aligned}
$$

and

$$
dG_t\left(t,z\right)=\frac{\lambda}{c}\frac{h\left(t,z\right)\int_{-\infty}^{\infty}xh(t,x)dx-zh\left(t,z\right)\int_{-\infty}^{\infty}h(t,x)dx}{\left(\int_{-\infty}^{\infty}h(t,x)dx\right)^2}dz.
$$

Now, we can substitute in expression (33) the above expressions for $G_t\left(t,z\right)$ and $dG_t\left(t,z\right)$,

to obtain:

$$
\begin{aligned}
z_0'(t) &\propto \int_{z_0(t)}^{\infty}zh\left(t,z\right)\left[\int_{-\infty}^{\infty}xh(t,x)dx-zx\int_{-\infty}^{\infty}h(t,x)dx\right]dz\\
&\quad+\int_{-\infty}^{z_0(t)}z_0(t)\left[h\left(t,z\right)\int_{-\infty}^{\infty}xh(t,x)dx-zh\left(t,z\right)\int_{-\infty}^{\infty}h(t,x)dx\right]dz
\end{aligned}
$$

$$
\propto\int_{z_0(t)}^{\infty}h\left(t,z\right)z\left[\frac{\int_{-\infty}^{\infty}xh(t,x)dx}{\int_{-\infty}^{\infty}h(t,x)dx}-z\right]dz+z_0(t)\int_{-\infty}^{z_0(t)}h\left(t,z\right)\left[\frac{\int_{-\infty}^{\infty}xh(t,x)dx}{\int_{-\infty}^{\infty}h(t,x)dx}-z\right]dz.
$$

Letting $\hat{z}(t)=\int_{-\infty}^{\infty}xh(t,x)dx\;/\;\int_{-\infty}^{\infty}h(t,x)dx$, we rewrite

$$
\begin{aligned}
z_0'(t) &\propto \frac{\int_{z_0(t)}^{\infty}zh\left(t,z\right)\left[\hat{z}(t)-z\right]dz}{\int_{-\infty}^{\infty}h(t,z)dz}+z_0(t)\frac{\int_{-\infty}^{z_0(t)}h\left(t,z\right)\left[\hat{z}(t)-z\right]dz}{\int_{-\infty}^{\infty}h(t,z)dz}\\
&< z_0(t)\frac{\int_{z_0(t)}^{\infty}h\left(t,z\right)\left[\hat{z}(t)-z\right]dz}{\int_{-\infty}^{\infty}h(t,z)dz}+z_0(t)\frac{\int_{-\infty}^{z_0(t)}h\left(t,z\right)\left[\hat{z}(t)-z\right]dz}{\int_{-\infty}^{\infty}h(t,z)dz}=0,
\end{aligned}
$$

where the final equality holds by definition of $\hat{z}(t)$.

Once concluded that $z_0'(t) < 0$, it immediately follows that $m_t(z,t) > 0$ for all $z \geq z_0(t)$, and this implies that $v'(t) < 0$. Now, let $\bar{v}(z, \hat{m}) = \frac{\lambda}{r+\lambda\hat{m}} z$ be the per-researcher expected discounted value of an innovation of value z, when a mass $\hat{m}$ of researchers are permanently engaged on the R&D line. When $c < \bar{v}(z, \hat{m})$, The cost of deploying an additional researcher on the considered R&D line cannot be recovered. Because $m_t(z,t) > 0$, it follows that the value $\bar{v}(z, m(z,t))$ decreases in t. Hence, there exists a time $T(z)$, after which R&D firms do not engage researchers in any R&D line of innovation value z any longer. Solving the equation $c = \frac{\lambda}{r+\lambda m(z,t)} z$, with $m(z,t) = \frac{z - z_0(t)}{c}$, we obtain the expression $z_0(T) = rc/\lambda$, reported in the statement of the Proposition. At that time T, it is also the case that $v(T) = \bar{v}(z, m(z,T))$: the equilibrium value function v is smoothly pasted with the function $\bar{v}(z, m(z, \cdot))$ for any innovation value $z \geq z_0(T)$. ■

Proof of Proposition 6. We rewrite equations (17) and (19) more succinctly as follows:

$$m_z(z,t) = -\frac{\lambda(m(z,t))}{\lambda'(m(z,t))} \frac{1}{z}, \tag{34}$$

$$\tilde{m}_z(z,t) = -\frac{\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)}{2\lambda'(\tilde{m}(z,t)) + \lambda''(\tilde{m}(z,t))\tilde{m}(z,t)} \frac{1 - \tilde{v}_z(t,z)}{z - \tilde{v}(t,z)}. \tag{35}$$

We first note that $1/z \geq (1 - \tilde{v}_z(t,z))/(z - \tilde{v}(t,z))$. In fact, this inequality is equivalent to $z\tilde{v}_z(t,z) \geq \tilde{v}(t,z)$, which holds because $\tilde{v}_{zz} \geq 0$.

Second, when the elasticity $\epsilon(\hat{m}) = -\hat{m}\lambda'(\hat{m}) / \lambda(\hat{m})$ of $\lambda(\hat{m})$ weakly increases in $\hat{m}$, the function $[\lambda(\hat{m}) + \lambda'(\hat{m})\hat{m}]/\lambda(\hat{m}) = 1 - \epsilon(\hat{m})$ weakly decreases in $\hat{m}$, and hence

$$\lambda(\tilde{m}(z,t)) \left[2\lambda'(\tilde{m}(z,t)) + \lambda''(\tilde{m}(z,t))\tilde{m}(z,t)\right] - \left[\lambda(\tilde{m}(z,t)) + \lambda'(\tilde{m}(z,t))\tilde{m}(z,t)\right] \lambda'(\tilde{m}(z,t)) \geq 0.$$

As a consequence, the first term in equation (34) is larger than the first term in equation (35) —note that both of them are negative, because $\lambda'(\tilde{m}(z,t)) < 0$ and $D_{\hat{m}}(\lambda(\hat{m}) + \lambda'(\hat{m})\hat{m}) < 0$ as $\lambda(\hat{m})\hat{m}$ is strictly concave.

In sum, the derivative m_z of the equilibrium allocation function m —calculated in expression (34)— is larger than $\tilde{m}_z$, the derivative of the optimal allocation function $\tilde{m}$, calculated in expression (35), which proves the stated result, because both functions satisfy the same resource constraints. ■

Proof that $\bar{v} = (\lambda/r)\, z_0$, and derivation of expressions (24) and (26). In order to prove that $\bar{v} = (\lambda/r)\, z_0$, we consider the expression:

$$\bar{v} = \int_0^\infty e^{-rt} \left[\frac{z}{m(z)} + \bar{v} - c\right] m(z)\, \lambda e^{-m(z)\lambda t} dt = \frac{\lambda m(z)}{r + \lambda m(z)} \left(\frac{z}{m(z)} + \bar{v} - c\right),$$

where $\bar{v} - c$ is the value for redeploying researchers once the innovation is discovered. Simplifying, we obtain: $\bar{v} = (\lambda/r)\left[z - m(z)\, c\right] = (\lambda/r)\, z_0$, for all $z \geq z_0$.

Plugging the solution $\tilde{m}(z)$ in the program (22), we solving for the optimal value $\tilde{v}(z)$, and obtain:

$$\tilde{v}(z) = \frac{\lambda\tilde{m}(z)\left[z - \tilde{m}(z)c\right] - \tilde{m}(z)u}{r + \lambda\tilde{m}(z)}. \tag{36}$$

Substituting this optimal value $\tilde{v}(z)$ into the first-order conditions (23), we obtain

$$\lambda\left[z-\frac{\lambda\tilde{m}(z)\left[z-\tilde{m}(z)c\right]-\tilde{m}(z)u}{r+\lambda\tilde{m}(z)}-2\tilde{m}(z)c\right]=u.$$

Solving for u and simplifying, we derive expression (24).

Expression (26) follows by first solving equation (24) for $\tilde{m}(z)$ to obtain that:

$$\tilde{m}(z)=(r/\lambda)\left(\sqrt{\frac{\lambda z-u}{rc}+1}-1\right),\ \text{ for all } z\geq z_0,$$

and then by noting that researchers are a perfectly divisible factor in our model, so that $\tilde{m}(z)=0$ at $z=z_0$, and, hence, $u=\lambda z_0$. ■

Proof of Proposition 8. We begin by calculating the aggregated welfare $W(m)$ associated with any allocation function m and associated density g.

The flow of aggregate welfare is expressed as:

$$rW(m)=\int_{z_0}^{\infty}\lambda m(z)\left[z-m(z)c\right]g(z)\,dz.$$

Each innovation of value z is discovered at arrival rate $\lambda m(z)$, upon discovery it accrues value z to the aggregate welfare but induces the aggregate cost $m(z)c$ as $m(z)$ researchers need to be allocated to different R&D lines.

Substituting in the expression $m(z)g(z)=(\alpha/\lambda)f(z)$, and rearranging, we obtain:

$$rW(m)=\alpha\int_{z_0}^{\infty}zf(z)\,dz-\alpha c\int_{z_0}^{\infty}m(z)f(z)\,dz. \tag{37}$$

Now, we consider the equilibrium allocation function $m(z)=(z-z_0)/c$, so the second term in the expression (37) takes the form:

$$\alpha c\int_{z_0}^{\infty}m(z)f(z)\,dz=\alpha\int_{z_0}^{\infty}(z-z_0)f(z)\,dz,$$

which, substituted back into the expression (37), gives

$$rW(m)=\alpha z_0\left[1-F(z_0)\right]. \tag{38}$$

Integrating condition (20) across z, we obtain the expression $\lambda M=\alpha[1-F(z_0)]$, that we substitute into expression (38), so as to obtain expression (27) for the aggregate equilibrium welfare.

We now consider the aggregate welfare $W(\tilde{m})$ associated with the optimal allocation $\tilde{m}$. Substituting the expression (26) of $\tilde{m}$ in the second term expression (37), and using $\lambda M=\alpha[1-F(z_0)]$, we obtain:

$$\alpha c\int_{z_0}^{\infty}m(z)f(z)\,dz=(\alpha/\lambda)\int_{z_0}^{\infty}\sqrt{c\lambda r(z-z_0)+c^2r^2}\cdot f(z)\,dz-rcM.$$

Further simplification leads to the aggregate optimal welfare expression (28). ■

The issues of monotonicity and convexity of the optimal value function $\tilde{v}_z\left(t,z\right)$.

In order to prove Proposition 5, we need to have that $\tilde{v}_z\left(t,z\right) \geq 0$ (at least for the case of hazard rate λ constant across R&D lines). In words, we need that the social value of researching undiscovered innovations increases in their value z. Likewise, in order to prove Proposition 6, we need to have that $\tilde{v}_{zz}\left(t,z\right) \geq 0$ (at least for the case of $c = 0$), the social value of researching undiscovered innovations needs to increase faster than linearly in their value z.

Neither of these results seem easy to achieve. Let us consider the first one.

Substituting $\tilde{m}(z,t)$ in the program (14) we obtain the flow value equation

$$[r + \lambda\tilde{m}(z,t)]\tilde{v}\left(t,z\right) = \lambda\tilde{m}(z,t)\left[z - \tilde{m}(z,t)c\right] - u\left(t\right)\tilde{m}(z,t) + \tilde{v}_t\left(t,z\right),$$

solving for the value, we get

$$\tilde{v}\left(t,z\right) = \frac{\lambda\tilde{m}(z,t)\left[z - \tilde{m}(z,t)c\right] - u\left(t\right)\tilde{m}(z,t) + \tilde{v}_t\left(t,z\right)}{r + \lambda\tilde{m}(z,t)}.$$

We differentiate $\tilde{v}$ using the Envelope theorem, we obtain:

$$\tilde{v}_z\left(t,z\right) = \frac{\lambda\tilde{m}(z,t) + \tilde{v}_{tz}\left(t,z\right)}{r + \lambda\tilde{m}(z,t)},$$

so as to get the equation:

$$\tilde{v}_z\left(t,z\right)r + \lambda\tilde{m}(z,t)\left[\tilde{v}_z\left(t,z\right) - 1\right] - \tilde{v}_{tz}\left(t,z\right) = 0. \tag{*}$$

Integrating the differential equation (16), we also obtain:

$$\tilde{m}\left(z,t\right) = \int_{\tilde{z}_0}^{z} \frac{1 - \tilde{v}_z\left(t,q\right)}{2c}dq = \frac{z - \tilde{z}_0}{2c} - \frac{\tilde{v}\left(t,z\right) - \tilde{v}\left(t,\tilde{z}_0\right)}{2c}.$$

But even after simplification of $\tilde{z}_0$ and $\tilde{v}\left(t,z_0\right)$, substituting back $\tilde{m}\left(z,t\right)$ in the above equation (*) we obtain a (possibly nasty) PDE. So, I stopped here.

Direct calculation of derivatives does not seem conclusive either.

From

$$\tilde{v}_t\left(t,z\right) = \frac{\tilde{v}_{tt}\left(t,z\right) - u'\left(t\right)\tilde{m}(z,t)}{r + \lambda\tilde{m}(z,t)},$$

we obtain:

$$\begin{aligned}\tilde{v}_{tz}(t,z) &= \frac{\begin{array}{c}[\tilde{v}_{ttz}(t,z)-u'(t)\,\tilde{m}_z(z,t)]\,[r+\lambda\tilde{m}(z,t)]\\ -[\tilde{v}_{tt}(t,z)-u'(t)\,\tilde{m}(z,t)]\,\lambda\tilde{m}_z(z,t)\end{array}}{[r+\lambda\tilde{m}(z,t)]^2}\\ &= \frac{[r+\lambda\tilde{m}(z,t)]\,\tilde{v}_{ttz}(t,z)-[ru'(t)+\tilde{v}_{tt}(t,z)\,\lambda]\,\tilde{m}_z(z,t)}{[r+\lambda\tilde{m}(z,t)]^2}\end{aligned}$$

which does not seem to lead anywhere.

It may be more fruitful to forward calculate $\tilde{v}(t,z)$ and then differentiate it using the envelope theorem.

Proving that $\tilde{v}_{zz}(t,z)\geq 0$ seems equally hard. The flow value equation is here

$$[r+\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)]\tilde{v}(t,z)=\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)z-u(t)\,\tilde{m}(z,t)+\tilde{v}_t(t,z),$$

which yields:

$$\tilde{v}(t,z)=\frac{\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)z-u(t)\,\tilde{m}(z,t)+\tilde{v}_t(t,z)}{r+\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)},$$

and

$$\tilde{v}_z(t,z)=\frac{\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)+\tilde{v}_{tz}(t,z)}{r+\lambda(\tilde{m}(z,t))\,\tilde{m}(z,t)}.$$

The problem seems to arise because the continuous innovation values formulation buries all (flow) continuation values in the time-derivative $\tilde{v}_t(t,z)$. The earlier two-value formulation, instead made continuation values explicit, and made it easier to understand and prove the results analogous to Proposition 5 and 6.

Consider the case of costly switching across R&D lines, without duplicative effort, return to the simple environment of section 3 and consider only two R&D lines, of values z_1 and z_2 with $z_1<z_2$. Let us simplify the exposition by assuming that $c\leq z_1P'(M)$, and note that the latter term is strictly smaller than $z_1P(M)/M=z_1\lambda/(r+M\lambda)$ because $P''<0$. In words, we assume that the deployment cost c is smaller than the marginal expected profit for participating in the least valuable race, 1, when all other R&D firms engage in that race.[48]

[48]This assumption guarantees that it is always socially optimal (and a fortiori profitable) to redeploy the researchers employed in either of the two R&Ds line $j=1,2$, when the innovation j is discovered, to the other R&D line, until both innovations have been discovered.

Due to the stationarity of the problem, it is never the case that researchers move across R&D lines in equilibrium, nor that it is optimal to redeploy them, unless one of the two R&D lines $j = 1, 2$ is exhausted as a consequence of innovation discovery. So, we can approach again the problem using standard dynamic programming techniques. Proceeding backwards, we first suppose that one innovation j has been discovered. At the time of discovery, of course, it is optimal and sequentially rational that all researchers are redeployed to the other R&D line, $k \neq j$. The value $\bar{v}_k$ of researching the innovation line k when all other M researchers are also engaged in line k is expressed as follows:

$$\begin{aligned} \bar{v}_k &= \int_0^\infty e^{-rt} \frac{z_k}{M} \left(M\lambda e^{-M\lambda t} \right) dt \\ &= \frac{\lambda}{r + M\lambda} z_k, \end{aligned} \tag{39}$$

because the innovation k arrives with aggregate hazard rate $M\lambda$ to one among the mass of M researchers employed, and each of them is the winner of the race with chance $1/M$, thus earning the value z_k.

Turning to the first stage of the game, in which both innovations are yet to be discovered, and denoting again by m_j the mass of researchers engaged in R&D line j, for $j = 1, 2$, we express the equilibrium value $v_j(m_j)$ of engaging in R&D line j through the Belman equation:

$$rv_j(m_j) = m_j \lambda \left[\frac{z_j}{m_j} + \max\{0, \bar{v}_k - c\} - v_j(m_j) \right] + m_k \lambda \left[\bar{v}_j - v_j(m_j) \right]. \tag{40}$$

Using the assumption that $c \leq z_1 \lambda / (r + M\lambda) = \bar{v}_k$, simplifying equation (40), rearranging it, and using $m_1 + m_2 = M$, we obtain:

$$v_j(m_j) = \lambda \frac{z_j - cm_j + m_1 \bar{v}_2 + m_2 \bar{v}_1}{r + \lambda M}. \tag{41}$$

Focusing again on interior equilibria as in section 3, we suppose that $0 < \{m_1, m_2\} < M$. Then, again, a no-arbitrage condition holds in equilibrium: it must be that $v_1(m_1) = v_2(m_2)$. Simplifying, we obtain the simple condition:

$$c(m_2 - m_1) = z_2 - z_1. \tag{42}$$

The equilibrium shares m_1 and m_2 are set so that the net expected individual gain $z_2 - z_1$ for engaging in the more valuable line 2 is exactly offset by the additional individual expected switching cost $c(m_2 - m_1)$; note that, here, the probability of losing the R&D race 1 or 2, and paying the redeployment c exactly equals m_1 and m_2.

Let us now turn to determining the optimal allocation $\tilde{\mathbf{m}} = (\tilde{m}_1, m_2)$. When neither innovation $j = 1, 2$ has been discovered, the Belman equation that characterizes the optimal aggregate welfare $W(\tilde{\mathbf{m}})$ is:

$$rW(\tilde{\mathbf{m}}) = \tilde{m}_1\lambda\left[z_1 + \max\{0, \bar{w}_2 - c\tilde{m}_1\} - W(\tilde{\mathbf{m}})\right] + \tilde{m}_2\lambda\left[z_2 + \max\{0, \bar{w}_1 - c\tilde{m}_2\} - W(\tilde{\mathbf{m}})\right] \tag{43}$$

where $\bar{w}_k$ is the social optimal value of redeploying all researchers to line $k \neq j$ after innovation $j = 1, 2$ has been discovered.[49]

Highlighting the differences with respect to the equilibrium Belman equation (40), we note the following. First, here, the innovation values z_1 and z_2 are not divided by $\tilde{m}_1$ and $\tilde{m}_2$ respectively, unlike in equation (40). The value of the discovery of innovation j is z_j for the society, regardless of the individual value which depends on the identity of the innovator. Second, the cost borne in the aggregate by switching $\tilde{m}_j$ researchers from R&D line j to line k when innovation j is discovered is $c\tilde{m}_j$, while the analogous individual cost was c in the equilibrium Belman equation (40).

Rearranging the Belman equation (43), and noting that for both $k = 1, 2$, it is the case that $c \leq \bar{w}_k/\tilde{m}_k = \bar{v}_k M/\tilde{m}_k$, as the latter is larger than $\bar{v}_k$, we obtain the following expression for the optimal welfare:

$$W(\tilde{\mathbf{m}}) = \lambda \frac{\tilde{m}_1\left[z_1 + \bar{w}_2 - c\tilde{m}_2\right] + \tilde{m}_2\left[z_2 + \bar{w}_1 - c\tilde{m}_1\right]}{r + \lambda M}.$$

By equating the first-order conditions associated with maximization of $W(\tilde{\mathbf{m}})$ under the constraint $\tilde{m}_1 + \tilde{m}_2 = M$, we obtain the condition:

$$2c(\tilde{m}_2 - \tilde{m}_1) = (z_2 - z_1)\left(1 - \frac{M\lambda}{r + M\lambda}\right), \tag{44}$$

[49] Note that $\bar{w}_k = \int_0^\infty e^{-rt} z_k \left(M\lambda e^{-M\lambda t}\right) dt = \frac{\lambda M}{r + M\lambda} z_k = M\bar{v}_j$: the social value $\bar{w}_k$ equals the private value $\bar{v}_j$ times the mass of employed researchers M.

which characterizes the interior solutions $\tilde{\mathbf{m}}$, for which $0 < \{\tilde{m}_1, \tilde{m}_2\} < M$.

By comparing the optimal condition (44) with the no-arbitrage equilibrium condition (42), we see that the social net expected gain for engaging an additional researcher in the hot R&D line 2, $(z_2 - z_1)\left(1 - \frac{M\lambda}{r+M\lambda}\right)$, is smaller than the net individual expected gain $z_2 - z_1$. Eventually, the society achieves both innovations, but engaging an extra researcher in the hot patent race 2 makes it marginally more likely that 2 arrives before than 1. Hence, the net benefit $z_2 - z_1$ is multiplied by $1 - \frac{M\lambda}{r+M\lambda}$, which is the expected discount factor associated with the time that lapses between the discovery of the first innovation and that of the second one. This difference induces an 'anticipation effect' that pushes towards overinvestment in the hot line 2, in equilibrium.

At the same time, the social marginal cost for engaging an additional researcher in the hot line, $2c\,(\tilde{m}_2 - \tilde{m}_1)$, is twice the private additional expected cost $c(\tilde{m}_2 - \tilde{m}_1)$ paid by the individual researcher. On top of this private cost, in fact, the society incurs also the extra deployment cost paid in expectation by all researchers already engaged in the hot R&D line 2, in case the additional researcher wins the hot R&D race for innovation 2. This 'deployment cost externality' also pushes towards equilibrium overinvestment in the hot R&D line 2.

Because the two effects we identified reinforce each others, we obtain again that $m_1 < \tilde{m}_1 < \tilde{m}_2 < m_2$, so that R&D firms overinvest in the more valuable line 2.

However, it is important to notice that, whenever c is small enough, it will be the case that neither the equilibrium and optimal solutions are interior, as can be seen by inspection of no-arbitrage condition (42) and the optimality condition (44). In this case, all researchers will be first engaged in the most valuable R&D line 2, in equilibrium. When innovation 2 is discovered, they will all be redeployed to the less valuable R&D line 1, until also innovation 1 is discovered. Most importantly, this unique equilibrium outcome is also socially optimal.

The above analysis is summarized in the following proposition.

Proposition 9 *In equilibrium of the dynamic model with* 2 *R&D lines and cost* c *of redeploying researchers across R&D lines, firms overinvest in the hot R&D line* 2 *whenever the* *cost c is larger than a given threshold* $\bar{c} > 0$.[50] *If* $0 < c < \bar{c}$, *then all researchers are initially engaged in the hot R&D line* 2, *and when innovation* 2 *is discovered, they are all redeployed to R&D line* 1, *until also that innovation is discovered; and this outcome is socially optimal.*

Now, consider the case of duplicative effort and costless switching of researchers across R&D lines, again with two R&D lines only.

Obvious modifications in the formulation and analysis of expression (42) imply that in an interior equilibrium with $0 < \{m_1, m_2\} < M$, here:

$$\lambda(m_1)z_1 = \lambda(m_2)z_2,$$

or, rearranging:

$$\frac{\lambda(m_1)}{\lambda(m_2)} = \frac{z_2}{z_1}. \tag{45}$$

The study of the optimal solution $\tilde{\mathbf{m}} = (\tilde{m}_1, \tilde{m}_2)$ is slightly more complicated. From the expression

$$rW(\tilde{\mathbf{m}}) = \tilde{m}_1\lambda(\tilde{m}_1)\left[z_1 + \bar{w}_2 - W(\tilde{\mathbf{m}})\right] + \tilde{m}_2\lambda(z_2)\left[z_2 + \bar{w}_1 - W(\tilde{\mathbf{m}})\right],$$

we obtain:

$$W(\tilde{\mathbf{m}}) = \frac{\tilde{m}_1\lambda(\tilde{m}_1)(z_1 + \bar{w}_2) + \tilde{m}_2\lambda(\tilde{m}_2)(z_2 + \bar{w}_1)}{r + \tilde{m}_1\lambda(\tilde{m}_1) + \tilde{m}_2\lambda(\tilde{m}_2)}.$$

At any interior solution, $0 < \{\tilde{m}_1, \tilde{m}_2\} < M$, the first-order conditions are:

$$\begin{aligned}
\mu &= \left[\lambda(\tilde{m}_1) + \lambda'(\tilde{m}_1)\tilde{m}_1\right](z_1 + \bar{w}_2)\left[r + \tilde{m}_1\lambda(\tilde{m}_1) + \tilde{m}_2\lambda(\tilde{m}_2)\right] \\
&\quad - \left[\tilde{m}_1\lambda(\tilde{m}_1)(z_1 + \bar{w}_2)\right]\left[\lambda(\tilde{m}_1) + \lambda'(\tilde{m}_1)\tilde{m}_1\right] \\
&= \left[\lambda(\tilde{m}_1) + \lambda'(\tilde{m}_1)\tilde{m}_1\right](z_1 + \bar{w}_2)\left[r + \tilde{m}_2\lambda(\tilde{m}_2)\right],
\end{aligned}$$

and

$$\mu = \left[\lambda(\tilde{m}_2) + \lambda'(\tilde{m}_2)\tilde{m}_2\right](z_2 + \bar{w}_1)\left[r + \tilde{m}_1\lambda(\tilde{m}_1)\right],$$

where μ is the Lagrange multiplier of the resource constraint.

[50] For consistency, it must be that $\bar{c} < z_1\lambda/(r + M\lambda)$. In the appendix, we prove that this is the case as long as $M > \frac{r}{2\lambda}\frac{z_2 - z_1}{z_1}$, which is a not a demanding condition.

Equating the first order conditions and rearranging, we obtain:

$$\frac{\lambda(\tilde{m}_1) + \lambda'(\tilde{m}_1)\tilde{m}_1}{\lambda(\tilde{m}_2) + \lambda'(\tilde{m}_2)\tilde{m}_2} = \frac{z_2 + \bar{w}_1}{z_1 + \bar{w}_2} \cdot \frac{r + \tilde{m}_1\lambda(\tilde{m}_1)}{r + \tilde{m}_2\lambda(\tilde{m}_2)}. \tag{46}$$

The comparison of equations (45) and (46) is simple and leads to the identification of two effects, both leading to the result that R&D firms overinvest in the more valuable hot line 2, i.e., that $m_1 < \tilde{m}_1 < \tilde{m}_2 < m_2$. The first effect is due to the fact that

$$\frac{r + \tilde{m}_1\lambda(\tilde{m}_1)}{r + \tilde{m}_2\lambda(\tilde{m}_2)} < 1, \quad \text{and} \ \frac{z_2 + \bar{w}_1}{z_1 + \bar{w}_2} < \frac{z_2}{z_1}.$$

The first inequality immediately follows from $\tilde{m}_1 < \tilde{m}_2$, because $\tilde{m}\lambda(\tilde{m})$ increases in $\tilde{m}$. To understand the second inequality, recall that

$$\bar{w}_k = M\bar{v}_k = \frac{\lambda\left(M\right)M}{r + \lambda\left(M\right)M} z_k,$$

and dropping the dependence on M in $\lambda(M)$ for simplicity, we have

$$\begin{aligned} \frac{z_2 + \bar{w}_1}{z_1 + \bar{w}_2} \quad &= \quad \frac{z_2 + \frac{\lambda M}{r + \lambda M} z_1}{z_1 + \frac{\lambda M}{r + \lambda M} z_2} - \frac{z_2}{z_1} = \frac{r z_2 + M\lambda z_1 + M\lambda z_2}{r z_1 + M\lambda z_1 + M\lambda z_2} - \frac{z_2}{z_1} \\ &= \quad (z_1 - z_2)\frac{M\lambda(z_1 + z_2)}{z_1\left(r z_1 + M\lambda z_1 + M\lambda z_2\right)} < 0 \end{aligned}$$

To gain some intuition, note that $\bar{w}_k$ is the optimal social value of the continuation after innovation $k = 1, 2$ has been discovered. This optimal continuation value is larger when the least valuable innovation 1 has been discovered, so that the more valuable innovation 2 is yet to be discovered. As a result, the social relative benefit $\frac{z_2 + \bar{w}_1}{z_1 + \bar{w}_2}$ for engaging in the most valuable R&D line 2 is smaller than the individual relative benefit z_2/z_1. Hence, the private incentive of joining the more valuable R&D line 2 is larger than the social incentive. This 'anticipation effect' is reinforced by a 'technological congestion' effect, when the elasticity $\epsilon\left(\hat{m}\right) = -\hat{m}\lambda'\left(\hat{m}\right)/\lambda\left(\hat{m}\right)$ of the arrival rate $\lambda\left(\hat{m}\right)$ weakly increases in $\hat{m}$. In fact, the function $\frac{\lambda(\hat{m}) + \lambda'(\hat{m})\hat{m}}{\lambda(\hat{m})}$ equals $1 - \epsilon\left(\hat{m}\right)$ and is thus decreasing when $\epsilon\left(\hat{m}\right)$ increases in $\hat{m}$. Thus, for both equations (45) and (46) to hold simultaneously when the elasticity $\epsilon\left(\hat{m}\right)$ does not decrease in $\hat{m}$, it must be that m_2 is smaller $\tilde{m}_2$ and m_1 is larger than $\tilde{m}_1$, so as to reduce $\lambda(m_2)/\lambda(m_1)$ relative to $[\tilde{m}_2\lambda'\left(\tilde{m}_2\right)) + \lambda\left(\tilde{m}_2\right))]/[m_1\lambda'\left(m_1\right)) + \lambda\left(m_1\right))]$.

The following propositions sums up our findings on the case of duplicative effort.

Proposition 10 *Consider a dynamic model with two R&D lines, costless redeployment of researchers across R&D lines, but with duplicative effort within each R&D line: the per-researcher discovery arrival rate* $\lambda(m_j)$ *of each innovation* j *at any time* t *decreases in the mass* m_j *of engaged researchers. If the function* $\lambda(\hat{m})\,\hat{m}$ *is increasing and concave in* $\hat{m}$*, and the elasticity* $\epsilon(\hat{m}) = -\hat{m}\lambda'(\hat{m})/\lambda(\hat{m})$ *does not decrease in* $\hat{m}$*, then R&D firms overinvest in the hot R&D lines, in equilibrium.*