Job Dynamics, Correlated Shocks and Wage Profiles*

Antonio Cabrales
Universitat Pompeu-Fabra
Hugo A. Hopenhayn
University of Rochester and Universitat Pompeu-Fabra

October 8, 1995

The last decade has witnessed a large body of empirical work which has substantively increased our knowledge about the facts concerning the performance of labor markets. One of the main features that arises from this literature is the large extent of labor reallocation that takes place in most developed economies, most of which is unrelated to sectoral, geographical or business cycle type shocks. This labor reallocation reflects the fact that opportunities do not arise always in the same places, so labor must be reallocated from less valuable matches to those that have become more valuable. One may thus conjecture that the performance of an economy must be closely affected by the relative ease with which this flow takes place, and thus by the policies that interfere with this reallocation process.

In recent years, the view that labor market restrictions in European economies may be responsible for high unemployment rates has been quite commonly held. Several theoretical models have been applied to provide a quantitative verdict. The results are far from clear. Bentolila and Bertola (1990) consider the effect of layoff costs on the hiring and firing decisions of firms which face idiosyncratic uncertainty to the returns to labor. They argue that if the shocks faced by firms are highly persistent and worker attrition high, the effect of this type of policy should be negligible. Hopenhayn and Rogerson (1993) incorporate this structure in a general equilibrium model, calibrating the stochastic process for firms' productivity to match US evidence on job creation and destruction. Their results suggest that layoff costs can reduce considerably the rate of turnover and thus the overall efficiency of the economy, but the effects on unemployment are undetermined. Finally, in a recent paper based on a labor matching model, Millard and Mortensen (1995) calibrate a specialized version of the Mortensen-Pissarides (1994) model and show that

*We would like to acknowledge the suggestions and help we had from Steve Millard and Dale Mortensen. The financial support of the Spanish Ministry of Education through DGYCIT grant PB93-0398 (Cabrales) is gratefully acknowledged.

layoff costs have a fairly small effect on turnover but can increase substantially the rate of unemployment.

The analysis of the effects of labor market policies thus appears to be highly model dependent. There are at least two reasons for this. Firstly, the process of wage formation has critical implications. In a situation where wages are exogenously given to the worker-firm match (as in the first two papers cited), layoff costs extend the duration of the job beyond the efficient level. On the contrary, if wages are renegotiated each period by the worker-firm pair and the negotiation is efficient, as in Millard and Mortensen, layoff costs -which increase the bargaining power of workers- have no direct effect on the decision of whether to terminate or not a match, but reduce the returns to creating new vacancies and thus contribute to higher unemployment rates. Secondly, as a compromise with analytical tractability, the models are very stylized and have introduced simplifying assumptions which may have non trivial implications for the quantitative analysis pursued. It thus become important to evaluate the robustness of this type of model to the particular structure assumed.

In this paper we will maintain the assumption of efficient bargaining in a matching model, but will attempt to overcome some of the empirical shortcomings of the model used in Millard and Mortensen. In particular, their model implies that hazard rates for job termination and average wages are independent of job tenure. In contrast, empirical work indicates quite robustly that hazard rates decrease and average wages increase with job tenure. The theory of job matching developed by Jovanovic was precisely motivated by this set of observations. We extend the Mortensen-Pissarides model along these lines. The probability structure introduced is almost identical to the one used in Hopenhayn (1992) to explain patterns of exit and growth of firms which are qualitatively identical to the ones corresponding to wage growth and job termination.[1]

Two types of policies are analyzed in the paper: wage taxes and layoff costs. Section **??** provides some theoretical results. The introduction of proportional wage taxes has some interesting implications for the bargaining process. When firm and worker have identical disagreement values, all wage taxes are paid by the worker. A somewhat modified version of this principle holds true in our extended Mortensen-Pissarides model. If the lifetime utility of an unemployed worker consists only of wage income which is subject to taxation, all taxes are absorbed by the worker and taxation has no real effects. If some component of the lifetime utility is not subject to this wage taxation, a real effect appears. We establish that the unemployment rate increases (decreases) with wage taxes provided that this additional component is positive (negative). The sign of the change in the unemployment rate is identical to the sign of the change in labor turnover.

We have argued above that the implications of policies on labor market behavior appear to be quite model dependent. In the paper we also show that they also depend on some

[1]This connection between labor and firm reallocation was recognized in Jovanovic (1982), that applies a version of the job matching model to study firm dynamics.

details of the policies which can be overlooked. In analyzing specific policies such as dismissal costs, the models described above have taken shortcuts which seem allegedly innocuous. As an example, though severance payments are typically a function (mostly linear) of seniority, the above models have taken it as a lump sum payment taken based on an average duration value. In the paper we show that this is assumption is not mere convenience, but bears directly into the issues analyzed. In particular, we establish that in our generalization of the Mortensen-Pissarides model, under quite general conditions linear severance payments are totally *undone* by the bargaining process and thus have no effect on aggregate employment. Another example on the importance of specific details of policy is illustrated by wage taxes. Lump sum wage taxes have no direct effect on match termination but proportional wage taxes do.

The paper is organized as follows. Section **??** describes the model. A stationary equilibrium is defined in section **??**. The theoretical results concerning wage taxes and some discussion of the effects of severance payments are discussed in section **??**. Section **??** provides a calibration of the extended Mortensen-Pissarides model and gives some numerical results. Section **??** develops the analysis of the effect of linear severance payments.

1 The model

The model we develop is a modified version of the one in Mortensen and Pissarides (1994) (subsequently MP). We study an equilibrium process of job creation and job destruction. Job creation results from a costly process of two-sided matching, where workers and employers engage in search activities. The jobs that are created after the search process is completed have a level of productivity that is random and match specific. Once established, the productivity of a match evolves according to an exogenous Markov process. As analyzed below, when a match reaches a low level of productivity, it is destroyed. Jobs are also continually created as displaced workers look for new jobs and entrepreneurs engage in new search to create positions.

The basic unit of the model is the worker-employer match, which we will also call job or position. This match produces a flow of output $q(x)$, which is strictly increasing in the productivity state of the match x. This state is match specific so if the match is dissolved the position becomes effectively destroyed.

The stochastic process for productivity x evolves as follows. The initial productivity of the match is drawn from some probability distribution, $G(x)$. This productivity is not known until the vacancy is filled. The evolution of productivity from that point on follows a Poisson jump process. New shocks arrive at a rate λ, and when they arrive the new value of the state is drawn from a probability distribution $F(x|x')$ where x' is the previous value of productivity. [2] Aside from the endogenous termination rule which will be discussed

[2] MP assume $F(x|x') = F(x)$. They also assume that $F(x)$ is a uniform distribution and $G(x)$ has

below, separations occur exogenously at rate δ.

Matches provide a flow of wages to the worker and profits to the firms, contingent on the productivity of the match x. We will denote by $w(x)$ the gross wage received by the worker, which is determined below. We include in the model payroll taxes, which are commonly used by countries to finance social security payments. These taxes are determined as proportions t_f of wages for firms and t_w for workers. Consequently, the net wage flow received by a worker is $w(x)(1-t_w)$ and the profit flow obtained from this match by the firm is $\pi(x) = q(x) - w(x)(1+t_f)$. We assume that both the owners of the firm and workers are risk neutral and discount future flows at a common interest rate r.

Entrepreneurs engage in search by posting a vacancy at a flow cost c per unit time. Matching occurs according to a constant returns to scale technology which is a function of V, the stock of posted vacancies and U, the stock of unemployed workers. Letting $\theta = V/U$, the matching technology can be described by a function $m(\theta)$, which measures the rate at which workers meet entrepreneurs. Letting $\hat{x}$ denote the minimum productivity level that makes initiating the match desirable[3], $e(\theta) = m(\theta)(1-G(\hat{x}))$ is the rate at which workers escape from unemployment and $1/e(\theta)$ the average duration of the unemployment spell. By the constant returns to scale assumption, $e(\theta)/\theta$ is the rate at which vacancies are filled.

The cost of terminating a match and the value of unemployment are affected by several labor market policies. Layoff compensations or severance payments are quite common among European economies.[4] Aside from this transfer, firms typically face other costs -such as those related to litigation and experience rating- when a worker is fired. Let T denote the total expected layoff cost to the firm. In what follows we will assume that all separations after the relationship is initiated, except the exogenous ones, are interpreted as firings. As indicated below, in the section about bargaining, this provides an upper bound on the possible effect of layoff costs. Aside from the compensation received by firms, workers receive while unemployed and for a certain length of time unemployment insurance transfers. We will denote by B the expected value of these transfers plus the layoff compensation received. Throughout this section B and T will be treated as exogenously given constants, which in our simulations correspond to some averages of the observed past compensations and layoff penalties, as in Millard and Mortensen (1995).

We assume that all separations between firms and workers are permanent. As we assume free entry and productivity is match specific, after separation a position has zero value

unit mass at the upper end of the support of $F(x)$.

[3] In the steady state -as is analyzed in the paper- this exit point is time independent. Notice that we are implicitly assuming that if a worker and a firm meet and the match is of quality inferior to $\hat{x}$ the relationship is not initiated, which implies that upon meeting the quality of the match is observed.

[4] In the UK it amounts to between 0.5 and 1.5 weeks per year worked, depending on the age of the worker. In Spain the law provides a compensation of 20 days per year worked. In some sectors, unions have negotiated even more favorable agreements.

to the firm. In contrast, since the supply of workers is fixed, an unemployed worker has a positive value at unemployment which is denoted by U.

Let $V(x)$ denote the value to a firm of having a match with productivity x and x^* be the level of productivity below which a match is terminated.[5]

This value satisfies the following dynamic programming equation:

$$rV(x) = q(x) - (1+t_f)w(x) + \lambda \left[\int_{x'>x^*} V(x')dF(x'|x) - F(x^*|x)T - V(x) \right] - \delta V(x) \quad (1)$$

The flow value $rV(x)$ for this firm consists of: *i)* the net profit flow; *ii)* the capital gain (or loss) associated to a change in productivity if the new productivity x' exceeds the cutoff level x^*; *iii)* the layoff cost T if the new shock is below x^* and *iv)* the loss of all value if the match is exogenously terminated.

In turn the value for the worker $W(x)$ satisfies the following dynamic programming equation:

$$rW(x) = (1-t_w)w(x) + \lambda \left[\int_{x'>x^*} W(x')dF(x'|x) + F(x^*|x)(U+B) - W(x) \right] + \delta(U - W(x)). \quad (2)$$

The interpretation of this equation is analogous to the one given for the firm's value. Notice that the value of an endogenous termination for the worker is $U + B$, consisting of the option value of unemployment plus the displacement transfers B. These transfers are assumed to be zero for an exogenous displacement.

The asset value of search for an unemployed worker is given by:

$$rU = b + m(\theta) \left[\int_{x'>\hat{x}} W(x')dG(x) + G(x^*)U - U \right] \quad (3)$$

The value V_e of posting a vacancy for a firm is given by

$$V_e = -c + \frac{m(\theta)}{\theta} \left[\int_{x'>\hat{x}} (V(x') - k)dG(x) \right] \quad (4)$$

where k is a training cost for the worker which is assumed to be paid entirely by the firm.

1.1 Bargaining process

The match specific nature of productivity creates at each instant a bargaining situation between entrepreneur and worker. The approach followed is that the wage rate must be such that total surplus will be divided in proportions that depend on the (exogenously

[5] x^* may be different from $\hat{x}$ because before the relationship is initiated the firm does not incur layoff compensations costs and after the match is created a destruction implies some costs for the firm.

given) bargaining power of the partners. This results from Nash's (1953) bargaining solution which maximizes the product of the utilities minus the values of termination of the relationship for the players. The proportion that goes to the worker is denoted β.

Nash (1953) considers a one-shot bargaining problem. The problem at hand is dynamic and we will assume that there is no commitment power by the agents, so wages can be renegotiated at any point in time. To make the solution time consistent, care has to be taken so that if the wage agreement today affects future bargaining positions, this effect is taken into account in the maximization. (This will become important when the severance payment depends on the length of the relationship.)

The Nash bargaining solution prescribes that the wage proposed in a given date and for a given productivity maximizes

$$(W(x) - (B+U))^{\beta} (V(x) + T)^{1-\beta} \tag{5}$$

This implies that

$$(1-\beta)(1+t_f)(W(x) - (B+U)) = \beta(1-t_w)(V(x)+T) \tag{6}$$

Equations (**??**) and (**??**) imply

$$(r+\lambda+\delta)(V(x)+T) = q(x) - (1+t_f)w(x) + (r+\delta)T + \lambda \left[\int_{x'>x^*} (V(x')+T) dF(x'|x) \right] \tag{7}$$

$$\begin{aligned} (r+\lambda+\delta)(W(x) - (B+U)) &= (1-t_w)w(x) - (r+\delta)(B+U) + \delta U && (8) \\ &\quad + \lambda \left[\int_{x'>x^*} (W(x') - (B+U)) dF(x'|x) \right] && (9) \end{aligned}$$

If we premultiply equation (**??**) by $\frac{\beta}{(1-\beta)} \frac{(1-t_w)}{(1+t_f)}$ subtract from it equation (**??**) and then rearrange we obtain:

$$w(x) = \frac{\beta(1-t_w)(q(x) + (r+\delta)T) + (1-\beta)(1+t_f)((r+\delta)(U+B) - \delta U)}{(1-t_w)(1+t_f)} \tag{10}$$

We have assumed that all separations are interpreted as firings, and equations (**??**), and (**??**) make it clear that this gives more bargaining power to the workers and a smaller expected value for firms. This leads to less job creation which is the channel through which layoff costs will lead to higher unemployment. Burda (1992) makes the opposite assumption, all separations that are not exogenous are interpreted as quits, and layoff costs (which in his model are only obtained after an exogenous separation) have no effects on unemployment since the layoff cost is considered by the bargaining pair as part of the workers' value and the wage is reduced accordingly.

In our model the assumption that separations are interpreted as firings is more sensible that in Burda's model because in our model, unlike in his, we have idiosyncratic match productivity. A worker that wants to quit and obtain the layoff compensation can lower the productivity level of the match below x^* and make the match undesirable enough that even paying the layoff compensation is justified.

2 Stationary Equilibrium

Matches are terminated when the state is such that total surplus is zero, The separation state x^* must then satisfy:

$$V(x^*) + T + W(x^*) - (U + B) = 0 \tag{11}$$

or given that (**??**) has to be satisfied

$$V(x^*) + T = 0. \tag{12}$$

Notice that provided that V and W are strictly increasing functions and that a solution to these equation exists, x^* and $\hat{x}$ will be uniquely determined.

The state $\hat{x}$ has to satisfy

$$V(\hat{x}) = 0. \tag{13}$$

Notice that if $V(x)$ is increasing $V(\hat{x}) = 0$ implies $\hat{x} \geq x^*$ and thus $W(\hat{x} \geq B + U \geq U$ so entering the relationship is also worthwile for the worker.

Implicit in this definition for $\hat{x}$ is the assumption that we do not allow for side payments prior to the establishment of the relationship. This implies that job creation decisions will be inefficient. A rule for efficient job creation would be to have

$$V(\hat{x}) + W(\hat{x}) - U = 0 \tag{14}$$

which would imply that $\hat{x} \leq x^*$ if $B - T > 0$, which is typically the case, since unemployment insurance enters in B, but not in T (unemployment insurance is usually not fully experience rated). To make (**??**) individually rational we would need a side payment $\alpha(x)$ that would make $V(x) + \alpha(x) \geq 0$ for $x \geq \hat{x}$.

We assume there are no side payments because we do not actually observe them. There may be several reasons for this. One is that there may be a problem of moral hazard, since employers with negative expected value may take the money and not hire the workers. Another is that workers may be credit constrained, and thus unable to come up with the money needed for the payment. This issue of side payment comes up again, in a modified fashion, in the discussion of time dependent layoff compensation. There, the initial wage works like the side payment. Since the layoff compensation depends on past wages one can set the initial wage low enough to compensate for future increases in bargaining power. A binding minimum wage in the context of time dependent layoff costs operates like the impossibility of initial side payments here and makes job creation inefficient.

Another assumption which we will use in the determination of an equilibrium is that there is free entry (except for the vacancy posting costs), new vacancies will be posted until

$$V_e = -c + \frac{m(\theta)}{\theta}\left[\int_{x'>\hat{x}} (V(x') - k) dG(x)\right] = 0 \tag{15}$$

Since $m(\theta)$ is an increasing function of θ, given values $V(x)$ and the exit rule, this equation uniquely determines θ.

In a stationary equilibrium the rate of match separation must equal the rate of match creation so that the total stock of matches remains constant. Exit can thus be thought of as a renewal process which switches positions that become unproductive or that are exogenously destroyed into new ones, sampled from the distribution G, conditional on the set $\{x > \hat{x}\}$. For any fixed separation rule x^*, and assuming the expectation of the stopping time associated to x^* is finite[6], this process has a unique invariant distribution over productivity states $\mu(x^*)$. Letting $\tau(x^*)$ denote the expected duration of the match associated to this separation rule and u the rate of unemployment, the hazard rate for match termination at the invariant distribution is $1/\tau(x^*)$ and thus the rate of escape of employment can be shown to be equal to $(1-u)/\tau(x^*)$.[7]In turn, the rate of match creation is given by $m(\theta)(1 - G(\hat{x}))u$.The requirement that creation flows balance out with match destruction flows implies that

$$(1-u)/\tau(x^*) = m(\theta)(1 - G(\hat{x}))u \tag{16}$$

A *stationary equilibrium* is given by the functions $V(x)$, $W(x)$ and $w(x)$, together with scalars U, x^*, θ and u that satisfy equations **??**, **??**, **??**,**??**, **??**,**??**, **??** and **??**.

2.1 Existence of a stationary equilibrium

The first step in showing that a stationary equilibrium exists is to show that the value functions are continuous and well defined.

Using the equilibrium condition (**??**) and the bargaining solution we have that

$$rU = b + m(\theta)\left[\frac{\beta(1-t_w)}{(1-\beta)(1+t_f)}\frac{c\theta}{m(\theta)} + \left((k+T)\frac{\beta(1-t_w)}{(1-\beta)(1+t_f)} + B\right)(1 - G(\hat{x}))\right] \tag{17}$$

[6]This condition is discussed in detail in Hopenhayn (1992)

[7]See Hopenhayn (1992) for details.

so the value of search while unemployed is a continuous function of θ and x^*, if $m(\theta)$ is continuous.

Lemma 1 Assume that the state of the match $x \in X$ where X is a metric space, and that $F(x'|x)$ has the Feller property. Then the $V(x)$ and $W(x)$ are well defined, continuous functions.

Proof:

We can write the operator that defines $V(x)$ as

$$(TV)(x) = \int_{x'>x^*} H(x, x', V(x'))dF(x'|x), \quad \forall x \in X \tag{18}$$

where $H(x, x', V(x')) = \frac{\lambda}{\lambda+r+\delta}((q(x) - (1 + t_f)\omega(x))/(1 - F(x^*|x)) + V(x') + T)$. $H : X \times X \times \Re \to \Re$ is a continuous function, which is also continuously differentiable in V and $H_3(x, x', y) = \frac{\lambda}{\lambda+r+\delta} < 1$. Since F has the Feller property, all the assumptions for Lemma 17.1 in Stokey, Lucas (1989) are satisfied and so the operator we use to define V has a unique fixed point in the space of continuous functions. Thus V is well defined and continuous. The proof for W is analogous.

Lemma 2 Under the conditions of Lemma 1, if $m(\theta)$ is a continuous function, and $X = [\underline{x}, \overline{x}]$, $\theta \in [0, \overline{\theta}]$ there are values of θ, $\hat{x}$ and x^* that solve (**??**) and (**??**) if $k = 0$ and

$$q(\overline{x})(1-\beta)-\beta(r+\delta)T-\frac{1+t_f}{1-tw}(1-\beta)\left[b+\overline{\theta}c\frac{\beta(1-t_w)}{(1-\beta)(1+t_f)}+(r+\delta)B\right]-\lambda T > 0 \tag{19}$$

$$q(\underline{x})(1-\beta)+\lambda\frac{q(\overline{x})(1-\beta)}{r+\delta}-\beta(r+\delta)T+\frac{1+t_f}{1-tw}(1-\beta)\left[b+(r+\delta)B\right] < -T(r+\lambda+\delta) \tag{20}$$

$$\frac{m(\overline{\theta})}{\overline{\theta}}\frac{q(\overline{x})(1-\beta)}{r+\delta} < c \tag{21}$$

.

Proof: We have shown that V, W and U are well defined continuous functions. Let the function $A1(\hat{x}, x^*, \theta) = \hat{x} - \min\{\max\{V(\hat{x}), \hat{x} - \overline{x}\}, \hat{x} - \underline{x}\}$, $A2(\hat{x}, x^*, \theta) = x^* - \min\{\max\{V(x^*) + T, x^* - \overline{x}\}, x^* - \underline{x}\}$ and $B(\hat{x}, x^*, \theta) = \theta + \min\{\max\{V_e - \theta\}, \overline{\theta} - \theta\}$ A fixed point of these mappings exists by Brouwer's fixed point theorem.

By equation (**??**) we have that

$$(\lambda + r + \delta)V(\overline{x}) > q(\overline{x}) - (1 + t_f)\omega(\overline{x}) - \lambda T \tag{22}$$

But by equation (**??**)

$$(\lambda + r + \delta)V(\overline{x}) > q(\overline{x})(1 - \beta) - \beta(r + \delta)T - \frac{1 + t_f}{1 - tw}(1 - \beta)\left[rU + (r + \delta)B\right] - \lambda T \tag{23}$$

and by equation (**??**) we have that

$$(\lambda+r+\delta)V(\overline{x}) > q(\overline{x})(1-\beta)-\beta(r+\delta)T-\frac{1+t_f}{1-tw}(1-\beta)\left[b+\overline{\theta}c\frac{\beta(1-t_w)}{(1-\beta)(1+t_f)}+(r+\delta)B\right]-\lambda T \tag{24}$$

Thus condition (**??**) guarantees that $V(\overline{x}) > 0$. Similarly equations (**??**) and (**??**) imply

$$(\lambda+r+\delta)V(\underline{x}) < q(\underline{x})(1-\beta)-\beta(r+\delta)T-\frac{1+t_f}{1-tw}(1-\beta)\left[b+(r+\delta)B\right]+\lambda V(\overline{x}) \tag{25}$$

Thus condition (**??**) guarantees that $V(\underline{x}) + T < 0$. Therefore conditions (**??**) and (**??**) guarantee by the definitions of $A1(\hat{x}, x^*, \theta)$ and $A1(\hat{x}, x^*, \theta)$ that a in a fixed point $\hat{x} \in (\underline{x}, \overline{x})$, and $x^* \in (\underline{x}, \overline{x})$, which implies that $V(\hat{x}) = 0$ and $V(x^*) + T = 0$. Condition (**??**) guarantees that for $\theta = \overline{\theta}$, $V_e < 0$ and $k = 0$ guarantes that for $\theta = 0$, $V_e > 0$, thus in a fixed point $\theta \in (0, \overline{\theta})$, which implies that $V_e = 0$. A fixed point, then, solves (**??**), (**??**) and (**??**).

The only thing that remains is to show that $\theta(x^*)$ is finite.

2.2 Wage and employment dynamics

In this section we provide conditions under which average wages increase and hazard rates for job destruction decrease as a function of job tenure. Notice that wages are strictly increasing in $q(x)$ and thus in the state x. This means that if the distribution of matches that survive t periods stochastically dominates the distribution of matches that survives $s < t$ periods, the average wage of workers with t years of tenure will be higher than the average wage of workers with $s < t$ years of tenure.

The following assumptions provide sufficient conditions for the distribution of matches of type x that have survived t years, $\mu_t(x)$ to dominate stochastically $\mu_s(x)$, when $s < t$.

A.1 a) $F(x|x')$ is decreasing in x' and jointly continuous.
b) for any x^*, $\lim_{x\to-\infty} F(x^*|x) = 1$.

A.2 G is continuous.

A.3 $\int F(x'|x)dG(s) \leq G(x')$ for all $s' \in \Re$.

A.4 i) $\forall y \in \Re$ and $x' \geq x''$, i) $[F(x'|x) - F(x''|x)]/[1 - F(x''|x)]$ is decreasing in x;
ii) $\int_{x'\geq x''} F(x''|x)dG(x) \leq G(x')[1 - F(x''|x)]/[1 - G(x'')]$.

Proposition 3 in Hopenhayn (1991) shows that under assumptions A.1 to A.4 the distribution of matches that survive from time s to time t conditional on any number of shocks

j_{st}, $\mu_t(x|j_{st})$ stochastically dominates $\mu_s(x)$ for all j_{st}. Therefore $\mu_t(x)$ stochastically dominates $\mu_s(x)$. [8]

This stochastic dominance result implies that wages increase and hazard rates for match destruction decrease with the length of job tenure.

Other properties of the equilibrium that are of interest concern the average values of firms and workers. Those are useful because when the model is calibrated one would like to match some moments of observables, and since the values of firms that are typically accessible are the averages (not the expectations) those are the ones whose match will be attempted, and we would like to know their relationship with other equilibrium values.

Let α_t be the probability that a match has at least a duration of t. The average value of a firm of age t is $\overline{V}_t = \alpha_t \int_{x'>x^*} V(x')d\mu_t(x')$. The average value of firms in the industry is thus $\overline{V} = \int_0^\infty \alpha_t \overline{V}_t / \int_0^\infty \alpha_t$. If $\mu_t(x)$ dominates stochastically $G(x)$ we have that $\overline{V} > \int_{x'>x^*} V(x')dG(x)$.

3 Policy Analysis

In this section we develop some theoretical results on the effect of wage taxes and layoff costs. Section **??** provides some simulations that suggest further implications of the model.

3.1 The effect of wage taxation

To understand the effect of wage taxes, it is instructive to consider first a static bargaining problem where the disagreement value is equal and normalized to zero for both, firm and worker. From equation **??** it follows that

$$w\left(x\right) = \frac{\beta q\left(x\right)}{\left(1+t_f\right)}. \tag{26}$$

Consequently, firm profits are given by

$$\pi\left(x\right) = q\left(x\right) - w\left(x\right)\left(1+t_f\right) = \left(1-\beta\right) q(x).$$

This simple result indicates that under Nash bargaining, proportional wage taxes are totally absorbed by the worker. Though this result does not generalize completely to the extended model, it is a good benchmark for the analysis.

We now turn to the more general case. To simplify the analysis we will assume that $B = T = 0$, that is that there are no layoff costs nor transfers and that there are no

[8] As shown in Hopenhayn (1992) conditions A.1 to A.4 are satisfied if $F(x'|x)$ follows an AR1 process $x' = \rho x + \epsilon$, where $0 < \rho < 1$ and ϵ is normally distributed with mean $\bar{\epsilon}$ and variance $\sigma_\epsilon^2 > 0$, and G is a normal distribution with mean $x_0 < \bar{\epsilon}/(1-\rho)$ and variance $\sigma_\epsilon^2/(1-\rho^2)$.

exogenous terminations ($\delta = 0$.) The disagreement values for firm and worker are then, respectively, 0 and U. We will use a subscript 0 to denote the equilibrium values when there is no taxation, i.e. when $t_f = t_w = 0$. Profits of the firm are given by

$$\pi(x) = (1-\beta)\left(q(x) - \frac{1+t_f}{1-t_w} rU\right) \tag{27}$$

In particular, this implies that $\pi_0(x) = (1-\beta)(q(x) - rU_0)$. Substituting in **??** we obtain

$$\pi(x) = \pi_0(x) + (1-\beta) r \left(U_0 - \frac{1+t_f}{1-t_w} U\right).$$

Hence the effect that introducing taxes has on the profits of the firm, depends on the difference

$$U_0 - \frac{1+t_f}{1-t_w} U. \tag{28}$$

To analyze this condition, it is instructive to consider the case where the value of leisure obtained while unemployed b is also zero. In this case, all utility received by the worker consists of wages received. Lets conjecture for this case that profits will not be affected by taxation. If such is the case, labor cost for the firm should be identical with and without taxation, and thus the equilibrium with taxes is identical to the one with no taxation except for a uniform lower level of consumption for workers. Consequently,

$$W(x) = W_0(x) \frac{1-t_w}{1+t_f}$$

and

$$U = U_0 \frac{1-t_w}{1+t_f}.$$

The rules for match separation would thus be identical with and without wage taxation, so the equilibrium with taxes implies that all taxes are effectively paid by the worker.

But this results holds only if all the sources of utility to the worker are subject to the same taxation. If there are some components which are not taxed, such as layoff compensation, unemployment insurance and the utility of leisure while unemployed, then

$$U > U_0 \frac{1-t_w}{1+t_f},$$

so $\pi(x) < \pi_0(x)$. In this case this "outside" income flow of the worker makes the firm absorb part of the tax. Consequently $V(x) < V_0(x)$, which by equation **??** implies that θ decreases and thus the unemployment rate increases. The separation rule in this case would change, leading to a higher turnover rate. The opposite result holds if the component that is not taxed has negative value for the worker. This would occur, for instance, if the worker dislikes considerably being unemployed. In this case both the rate of unemployment and turnover would decrease with the tax.

3.2 Effect of layoff compensation

In this section we consider the effect that layoff compensation can have on workers compensation, turnover and unemployment. To simplify the analysis, we consider the case where wage taxes are zero and there are no exogenous separations.

Consider a policy that introduces a severance payment Δ which is a transfer from the firm to the worker. This implies that T and B increase by Δ. A first order effect -leaving U constant- implies no change in the rule for job termination and an increase in wages at each state equal to $r\Delta$, which benefits directly all workers. As profits decrease, θ must also decrease and thus the unemployment rate increases. Unless the initial value of θ were inefficiently low, the final effect must be a reduction in the value of unemployment U. This in turn implies a reduction in the exit point x^* and consequently a lower rate of turnover.

Since $W(x^*) = B + U$, a worker at the lowest level will be better off provided that the decrease in U is less than Δ and will be worse off otherwise. In the latter case it could be in the interest of these workers to favor eliminating severance payments and thus a potential conflict of interests could arise between employed workers with different job quality. In our quantitative analysis, however, all employed workers appear to be better off with the taxes.

It is interesting to note that in this setup, all workers -employed and unemployed- would favor a policy that prescribes eliminating severance pay for all new contracts or similarly, allowing for temporary employment.[9] A casual reading of the evidence in Spain shows that trade unions have been consistently opposed to introducing such added flexibility in the labor market. The special result obtained in the context of this model occurs because unemployed workers place no competitive threat on employed workers.[10]

4 Quantitative Analysis

In this section we calibrate the model presented in the previous sections to US data in order to assess the effects of changes in the policy parameters on the equilibrium values for employment, turnover and wages. Our approach will be to try to follow the parametrization of Millard and Mortensen (1995) as close as possible, so that our results can be compared to theirs. This will provide a measure of the sensitivity of the model to the specific assumptions about the stochastic process for job productivity.

The time unit in the model is a quarter and all flows will be reported in per quarter terms.

[9] It may be arguable whether committing to such policy is actually credible. Once a sufficiently large critical mass of new unprotected workers appear, political pressures for reestablishing severance pay for all those employed could reappear.

[10] We are currently considering an extension to the MP framework to allow for such competition.

The rate of interest is $r = 0.01$. The exogenous rate of attrition (quit to unemployment), $\delta = 0.015$. The matching function $m(\theta) = \theta^{\eta}$, where $\eta = 0.6$. We normalize all values of productivity to a numeraire which will be the long run value of output. The training cost and recruiting cost figures must be interpreted in terms of this numeraire. The vacancy cost $c = 0.33$ and the training cost $k = 0.275$. The bargaining power of the worker is $\beta = 0.3$. The specific values chose are justified in Millard and Mortensen (1995).

We now consider the policy parameters. The social security taxes are $t_f = 0.075$ and $t_w = 0.075$. The number of quarters during which unemployment insurance is received is 2. The average replacement ratio of unemployment insurance is 25%, the product of the mandated replacement ratio, which is 50% and the percentage of people who qualify for the benefits, around 50%. Severance pay in the US is zero, but there is a cost of layoff since employers pay an average of about 60 cents for each additional dollar that former employees receive in the form unemployment insurance.

The values of the remaining parameters are chosen to obtain an adequate fit with values of unemployment duration, incidence and the existing estimates of the effects of tenure on wage profiles and hazard rates of termination of the employment.

In our calibration $\lambda = 0.25$. This implies the arrival of shocks in our model is more frequent than in Millard and Mortensen (1995) who use $\lambda = 0.1$. Notice that in our model, unlike in theirs, there are two measures of persistence of shocks, namely, λ the parameter that controls their rate of arrival and ρ the parameter that weights the last value of the shock in the autoregressive process that controls the value of the shock once it arrives. We also make the value of leisure, $b = 0$.

The stochastic process assumed for the shocks is an AR(1) with normal innovations. The persistence of the shocks we impose is $\rho = 0.95$. The mean of the innovation is 0.001. The variance of the innovation is 0.65. As for the initial value of productivity we assume a normal distribution with mean 0.004 and variance 0.25.

The benchmark values obtained for our simulations are shown in table A.1. A comparison with US data shows that the fit for the incidence of unemployment and duration are adequate. The qualitative picture of dynamics is as we predicted. A comparison of the evolution of wages with tenure and the rates of growth obtained from Topel (1991), reported on Table A.2 shows a remarkable approximation. The variance of wages grows with tenure (after the second year) which also squares with available evidence. Inspection of the invariant distribution shows that the distribution of wages is quite skewed. Hazard rates of separations are higher than one would expect from the estimates of McLaughlin (1991) but the qualitative features of decreasing hazards and decreasing rate of change are captured by the model.

Three policy experiments were performed and the results are reported in tables A.3., A.4 and A.5. Table A.3. reports the calibration for the economy where a severance payment of 0.27, around a month of mean wages. Consistently with the findings of Millard and

Mortensen (1995) the effect on unemployment is rather sizable, with an increase of about 2.2 percentage points. This effect comes largely through an increase in the duration of unemployment, since the effect on the rate of separations is negligible. Wages are higher as a reflection of the increased bargaining power of workers. The value of an employed worker, $W(s)$, shows that all employed workers benefit from the severance pay. The effect is negative for the firm whose value decreases and for the unemployed.

The second policy experiment (Table A.4) performed is given by a suppression of the payroll tax. Our results show an almost negligible decrease in the unemployment rate as the tax is eliminated. From the discussion given in section **??**, this occurs because the non taxed component of the worker's utility is zero. We actually used a value of zero for the flow utility received by a worker when unemployed to decrease the value of unemployment and thus reduce duration, which in our calibration was too high if we used a positive value of leisure. The turnover rate changes by a very small amount. In this case the productivity increases since less of it leaks to the government, and this induces large increases in value of around 6%. The qualitative dynamic figures are almost the same.

The third policy experiment (Table A.5) concerns the suppression of the experience rating for unemployment insurance. In this case the decrease of unemployment in our model is 0.8%. Turnover is virtually unaffected, so all of the change comes from a decrease in duration, 0.15 quarters (a little under two weeks). The same experiment in Millard and Mortensen (1995) induced a decrease of unemployment of only 0.25%, turnover went up by 0.53% and duration went down by 0.33 months (10 days). This example confirms that the assumptions about the stochastic process of shocks can have implications on the effects of policy even if several features of the baseline calibration, like the figures for unemployment rate, turnover and duration of unemployment are consistent with real data.

One conclusion from these experiments is that this type of models is rather sensitive to modelling details. The change in the value of leisure, and the specification of a different stochastic process for the shocks are not merely an instrument to match a few extra observables, like hazard rates of employment and wage profiles; they have implications on the effects of policies and therefore they should be considered carefully when specifying the model.

5 Time dependent layoff compensation

We have already observed that in many countries the legislation about severance pay establishes that its amount should be proportional to the length of time the worker was employed at the firm when the firing occurs. In the models of Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993), Millard and Mortensen (1995) as well as in our simulations of last section, a lump sum payment equal to the average observed payment, is used instead. We show here that this assumption is not innocuous and if used one should be aware of the bias it might introduce.

The first observation we make is that in the absence of taxes and when the severance pay is linear in tenure, the existence of severance pay has no real effects in the model. It only changes the shape of the wage profile, without affecting the present value. This is a consequence of the efficiency of Nash bargaining and the fact that the severance pay is a pure transfer. In the absence of distorting taxes termination is efficient and thus not affected by the severance payments. Since at the time the match is produced the accumulated severance payments are zero, they have no effect in how this total ex-ante surplus from the match is shared. This can be effected in spite of the fact that as time evolves wages must increase -due to the increase in the bargaining power of the worker-, by establishing lower wages at the outset. The worker and firm rationally forecast the effect of wages on future bargaining power and determine the division of the surplus with the appropriate initial wage.

We also establish that the linear severance pay causes wages to increase with tenure. This should not be surprising since as we mentioned above, the linear severance pay increases the bargaining power of the worker as the relationship matures. Our observation suggests as a hypothesis that countries or industries with higher severance payments should exhibit steeper wage profiles.

Proposition 1. Let x_0^*, θ_0, u_0, U_0, $V_0(x)$, $W_0(x)$ and $w_0(x)$ describe the steady state of the model when $B = T = t_f = t_w = 0$.[11] Suppose now $B = T = \alpha t w(t,x)$ where $\alpha > 0$ and $t_f = t_w = 0$. Let $V(t,x)$ and $W(t,x)$ be the equilibrium values of a match that is t periods old for the firm and worker respectively. Let $x^*(t)$ be the level of productivity at which the match separates as a function of seniority. There is an equilibrium for this model where $\theta = \theta_0$, $\hat{x} = x^*(t) = x_0^*$ for all t, $V(0,x) = V_0(x)$, $W(0,x) = W_0(x)$ for all x, $u = u_0$ and $U = U_0$.

Proof: See appendix.

¿From the proof of the proposition the time path of the values of the firm and the worker, and also the profile over time of wages along this equilibrium path are obtained. Substituting equation (**??**) into equations (**??**) and (**??**) we have a system of ordinary differential equations (2× the number of states for productivity) that give the value for the worker and the firm. The initial conditions for the system are $W_0(x)$ and $V_0(x)$

This implies that a differential equation determines the evolution of wages as well.

By equation (**??**) we have (letting $a_1 = \frac{(1-\beta)}{\beta}$), that

$$(1 + a_1)\alpha t\dot{\omega}(t,x) + (1 + a_1)\alpha\omega(t,x) = a_1\dot{W}(t,x) - \dot{V}(t,x) \tag{29}$$

[11]Since $B = T = 0$ the equilibrium value for $\hat{x} = x_0^*$

Equations (??), (??) and (??) give us

$$\begin{aligned} a_1\dot{W}(t,x) - \dot{V}(t,x) &= q(x) - (1+a_1)\omega(t,x) + (r+\lambda)(a_1 W(t,x) - V(t,x)) \\ &\quad -\lambda\left[\int_{x'>x_0^*}(a_1 W(t,x') - V(t,x'))dF(x'|x) - F(x_0^*|x)[(1+a_1)\omega(t,x_0^*)\alpha t + a_1 U]\right] \end{aligned}$$

substituting into (??)

$$\begin{aligned} (1+a_1)\alpha t\dot{\omega}(t,x)) &= [r(1+a_1)\alpha t - (1+a_1)(1+\alpha)]\,\omega(t,x) \\ &\quad +q(x) + ra_1 U + \lambda(1+a_1)\alpha t\omega(t,x) \\ &\quad -\lambda\left[\int_{x'>x_0^*}(1+a_1)\alpha t\omega(t,x')dF(x'|x) - F(x_0^*|x)(a_1+1)\omega(t,x_0^*)\alpha t\right]. \end{aligned}$$

Proposition 2: If $\lambda = 0$, $\omega(t,x)$ is increasing in t.

Proof: See appendix.

The proposition does not tell us what happens in the case where $\lambda > 0$. This case is more difficult to deal with because of the interaction of the dynamics of bargaining with the fact that the process for the shocks we assume is mean reverting. This mean reversion tends to make the expected wage increasing for people with a value of the state below the mean and decreasing for people with above the mean productivity. In the dynamic bargaining when the layoff costs are time dependent the firm and the worker have to take into account that an increase in the wage will have an impact in future bargaining power so if the state of the match is below average they may not want to raise the salary considering that the expected changes in the state may do that job and a wage increase at this point may lead to a too strong bargaining position for the worker once the change in productivity occurs. It may even be that they want to reduce the salary to compensate the future increases in bargaining power.

6 Conclusions

We have shown in this paper that it is easy to extend the model of Mortensen and Pissarides (1994) to account for several empirical observations in labor markets. The model is useful because it is manageable and it provides insights into the workings of the labor markets and the effects of policies, but the conclusions seem to be sensitive to the precise specification chosen. For this reason further research into these issues would be necessary to know the extent to which the conclusions here would need to be modified.

We would like to investigate the effects of introducing a type of temporary contract whose disolution would imply no layoff compensation, as happens for example in Spain. This type of contract implies no loss for already employed workers in our context, because it does not affect their effective bargaining position. However, one observes a very strong opposition of unions to their implementation, which suggests that there should be some form of implicit competition that this type of model does not capture. For this to happen, productivity would have to be only partially match dependent, not totally.

Another point that should be mentioned in this context is that since the quality of workers is only match dependent we cannot explain decreasing hazard rates for unemployment. It is probable that the reason why the long term unemployed are less likely to find employment is a sorting argument similar to the one we use to explain the hazard rates for employment. People with longer unemployment tend to be less skilled. We would need to have a quality variable for people that were somewhat independent of the match they are in.

It is interesting to note that turnover is almost unaffected by many changes in policy parameters. This is due to the fact that with Nash bargaining termination of the match is efficient and creation of matches is more affected than destruction. There is some separation between production and match technology. This feature would disappear if there were a resource that were used by both technologies, say in a general equilibrium model where capital could be used to improve the matching technology so that there were certain substitutability between production and matching.

A policy whose study has not been undertaken in this model is the establishment of a minimum wage. Its study has been absent because of difficulties in analytical tractability. The reason is that a minimum wage has an impact in bargaining, which in its presence has to take into account an additional restriction. The study of minimum wages is interesting in its own right and also because of the interaction with, for example, time dependent layoff compensation since it tends to offset the neutrality of layoff compensation that we proved in Proposition 1.

Another extension to the model would contemplate the state of the match as a result of learning about its intrinsic quality, not of shocks to productivity. That leads to a stochastic process for the state of the match that is a martingale.

Finally, it would be interesting to know what are the effects of policy in a context with risk averse agents. This is important because for some types of policy, like the unemployment insurance or the layoff compensation the insurance motive appears to be very important, and an adequate asessment of the impact of this policy should consider agents who are risk averse.

7 Allowing effort in search process

Each position has a productivity of x'. Shocks to productivity arrive at rate λ'. When a shock arrives, the new productivity is x with probability p and 0 with probability $1-p$. Thus, when a shock arrive the position is terminated if productivity becomes $x' = 0$. Let $\lambda = \lambda' p$. The discount rate is r.

Let V be the asset value of the position for the firm, and W the asset value for the worker. The stochastic structure of productivity implies that,

$$V + W = \frac{x}{r+\lambda} \tag{30}$$

We will assume that there is a layoff compensation, K, to be paid upon any termination. The courts cannot verify what was the productivity of the job upon termination, so if the worker and the firm do not agree on a wage and the worker leaves the firm the compensation has to be paid. With this threat point established, Nash bargaining implies that,

$$\beta(V + K) = (1-\beta)(W - U - K) \tag{31}$$

where U is the asset value of being unemployed.

The rate at which an unemployed workers meet firms is equal to $em(\theta)$, where θ is vacancies over unemployment, a measure of labor market tightness. Correspondingly, $em(\theta)/\theta$ is the rate at which vacancies are filled. The variable e represents the effort made by the worker in the intensity of search. When the worker searches more intensely the likelihood of finding a job is larger. It seems natural to assume that the e in the worker's rate is her own effort, while the e in the firm's rate is the average effort. However, since all worker's will be identical the individual effort will equal the average effort, so we will not distinguish them notationally. Search effort has a cost, which, for simplicity reasons, we model as a cuadratic function.

The flow benefit per unit of time for an unemployed worker is b. Thus, the equation that determines the value of being unemployed U is,

$$rU = b + em(\theta)(W - U) - \frac{1}{2}de^2 \tag{32}$$

The condition for optimality of the worker's effort is,

$$de = m(\theta)(W - U) \tag{33}$$

The value V_e of posting a vacancy for a firm is given by,

$$V_e = -c + e\frac{m(\theta)}{\theta}V \tag{34}$$

An assumption that will be used in determining an equilibrium is that there is free entry so that $V_e = 0$. This implies that

$$e\frac{m(\theta)}{\theta}V = c \tag{35}$$

Using equations (**??**), (**??**), (**??**), (**??**) and (**??**) we can show that

$$U = \frac{1}{r}(b + \frac{1}{2}de^2) \tag{36}$$

$$W = K + \beta\frac{x}{r+\lambda} + (1-\beta)\frac{1}{r}(b + \frac{1}{2}de^2) \tag{37}$$

$$V = -K + (1-\beta)\left(\frac{x}{r+\lambda} - \frac{1}{r}(b + \frac{1}{2}de^2)\right) \tag{38}$$

$$\frac{m(\theta)}{\theta} = \frac{c}{e(-K + (1-\beta)(\frac{x}{r+\lambda} - \frac{1}{r}(b + \frac{1}{2}de^2)))} \tag{39}$$

Assume that $m(\theta) = \theta^\gamma$, then e is determined implicitly by the following equation,

$$\left(\frac{e\left(-K + (1-\beta)\left(\frac{x}{r+\lambda} - \frac{1}{r}(b + \frac{1}{2}de^2\right)\right)}{c}\right)^{\frac{\gamma}{1-\gamma}} \left(K + \beta\frac{x}{r+\lambda} - \beta\frac{1}{r}(b + \frac{1}{2}de^2\right) = de \tag{40}$$

Solving (**??**) one can obtain numerically the equilibrium values of e and then the rest of the variables. One can also use the implicit function theorem to obtain several elasticities and derivatives.

Let us turn to the effect of the layoff compensation on the labor market tightness and the search effort. Intuition tells us, and the analysis bears out the intuition, that when the rate at which firms meet workers is not very sensitive to the vacancies to unemployment ratio (that is, when *gamma* is close to one, and the cost of effort d is low) the elasticity of vacancies over unemployment to the layoff cost is small. This happens because the layoff cost implies less bargaining power, thus less surplus, for firms. To preserve the zero profit condition it must be the case that firms find workers faster. This will happen either because there are more unemployed people per vacancy, or because the worker's search effort increases. Since adjusting the matching rate through changes in job market tightness is harder (because of the low elasticity) and the effort is cheap, worker's will look harder for a job.

However, when the cost of search (or the elasticity) are higher, one can find that the search effort decreases with the layoff cost and the elasticity of tightness to the layoff cost is higher. The reason is that in this case, as unemployment increases, the workers become discouraged easily because of the costliness of the search effort and they work less hard at finding a job, and this forces unemployment to be even higher to recover the incentive to invest in posting vacancies.

To see this through the equations notice that using equation (**??**) and due to the implicit function theorem,

$$\left(\left(\frac{\gamma}{1-\gamma}-1\right)\frac{(eV)^{\frac{\gamma}{(1-\gamma)}}}{e^2}(W-U)-(1-\beta)\frac{de}{r}e^{\frac{\gamma}{}-1}(W-U)-\beta\frac{de}{r}\frac{(eV)^{\gamma/(1-\gamma)}}{e}\right)\frac{\partial e}{\partial K}=$$
$$2K+(2\beta-1)\left(\frac{x}{r+\lambda}-\frac{1}{r}(b+\frac{1}{2}de^2)\right)$$

Suppose that β is close to 1/2 to make the arguments simpler. One can readily see that a value of γ close to one, or a low value of d makes the left hand side positive thus $\frac{\partial e}{\partial K}$ positive. When γ is close to zero or d is high we have $\frac{\partial e}{\partial K}$ negative.

Equation (**??**) tells us that

$$\theta^{-\gamma}\frac{\partial\theta}{\partial K}=-1+\frac{\partial e}{\partial K}\left(\frac{V}{c}-\frac{de^2}{r}\right) \tag{41}$$

Given what we said about $\frac{\partial e}{\partial K}$ it must be the case that a γ close to one and a small d decrease the elasticity of θ with respect to K and γ close to zero and d large increase that elasticity.

8 Appendix

Proof of Proposition 1. We check that under the assumptions of Proposition 1 the conditions for a steady state equilibrium are satisfied. The equation for the asset value of the position for the firm is now:

$$rV(t,x)=q(x)-\omega(t,x)+\lambda\left[\int_{x'>x^*(t)}V(t,x')dF(x'|x)-F(x^*(t)|x)\omega(t,x^*(t))\alpha t-V(t,x)\right]+\dot{V}(t,x) \tag{42}$$

The equation for the asset value of the position for the worker:

$$rW(t,x) = \omega(t,x) + \lambda \left[\int_{x' > x^*(t)} W(t,x') dF(x'|x) + F(x^*(t)|x)(\omega(t,x^*(t))\alpha t + U) - W(t,x) \right] + \dot{W}(t,x) \tag{43}$$

Nash bargaining prescribes that:

$$\beta(V(t,x) + \alpha t\omega(t,x)) = (1-\beta)(W(t,x) - \alpha t\omega(t,x) - U) \tag{44}$$

$$(1 + a_1)\alpha t\omega(t,x) = a_1(W(t,x) - U) - V(t,x) \tag{45}$$

The asset value of search for an unemployed is given by:

$$rU = b + m(\theta)\left[\int_{x' > \hat{x}} W(x',0) dG(x) + G(\hat{x})U - U\right] \tag{46}$$

This equation is satisfied for $\theta = \theta_0$, $\hat{x} = x_0^*$, $W(x,0) = W_0(x)$ and $U = U_0$, by definition of U_0.

A condition for equilibrium is that competition drives the value of search for a firm to zero, that is;

$$0 = -c + \frac{m(\theta)}{\theta}\left[\int_{x' > \hat{x}} (V(x',0) - k) dG(x)\right] \tag{47}$$

This condition is satisfied when $V(x',0) = V_0(x)$, $x^*(0) = x_0^*$ and $\theta = \theta_0$, since these are equilibrium values for the problem without severance pay.

Let $S(t,x) = V(t,x) + W(t,x) - U$ and $S_0(x) = V_0(x) + W_0(x) - U_0$. The other condition is the one that determines $x^*(t)$:

$$S\left(t, x^*\left(t\right)\right) = V(t, x^*(t)) + W(t, x^*(t)) - U = 0 \tag{48}$$

Since in the equilibrium we are trying to verify $S(x,t) = S_0(x)$ and x_0^* solves $S_0(x) = 0$, $x^*(t) = x_0^*$ is consistent with the proposed solution. $S(x,t) = S_0(x)$ also implies that $\dot{V}(t,x) + \dot{W}(t,x) = 0$. Noting this and adding up equations (**??**) and (**??**) we obtain;

$$(r + \lambda)S(t,x) = q(x) + \lambda\left[\int_{x' > x^*(t)} S(t,x') dF(x'|x)\right] - rU \tag{49}$$

With $U = U_0$ and $x^*(t) = x_0^*$ the equation is satisfied by definition of $S_0(x)$.

The only thing that remains to be checked is that it is consistent with the equilibrium conditions that $V(0,x) = V_0(x)$ and $W(0,x) = W_0(x)$ for all x. The two equations that determine $V(0,x)$ and $W(0,x)$ are the bargaining equation,

$$\beta V(0,x) = (1-\beta)(W(0,x) - U) \tag{50}$$

This is satisfied since $U = U_0$ and

$$\beta V_0(x) = (1-\beta)(W_0(x) - U_0) \tag{51}$$

Another equations to satisfy is: $W(0,x) + V(0,x) - U = S(0,x) = S_0(x)$, which is true since $W_0(x) + V_0(x) - U_0 = S_0(x)$.

And finally since $\hat{x} = x^*(t) = x_0^*$ for all t, $\theta = \theta_0$ and $u = u_0$ the invariant distribution for the process is the same and

$$(1-u)/\tau(x_0^*) = m(\theta)(1 - G(x_0^*))\,u \tag{52}$$

holds. This completes the proof.

Proof of Proposition 2. When $\lambda = 0$ we have by equation (**??**) that

$$\dot{\omega}(t,x) = \left(r - \frac{1+\alpha}{\alpha t}\right)\omega(t,x) + \frac{\beta q(x) + rU}{\alpha t} \tag{53}$$

integrating this expression we obtain,

$$\omega(t,x) = \exp\left[\int_0^t \left(r - \frac{1+\alpha}{\alpha s}\right) ds\right]\omega(x,0) + \int_0^t \exp\left[\int_s^t \left(r - \frac{1+\alpha}{\alpha u}\right) ds\right] \frac{\beta q(x) + rU}{\alpha s} ds \tag{54}$$

which implies

$$\omega(t,x) = \left(\int_0^t \exp(-rs) s^{1/\alpha} ds\right) \frac{\beta q(x) + rU}{\alpha} \frac{\exp(rt)}{t^{(1+\alpha)/\alpha}} \tag{55}$$

Equation (**??**) plus equation (**??**) imply

$$\dot{\omega}(t,x) = \left(r - \frac{1+\alpha}{\alpha t}\right)\left(\int_0^t \exp(-rs) s^{1/\alpha} ds\right) \frac{\beta q(x) + rU}{\alpha} \frac{\exp(rt)}{t^{(1+\alpha)/\alpha}} + \frac{\beta q(x) + rU}{\alpha t} \tag{56}$$

which implies that

$$\text{sign}(\dot{\omega}(t,x)) = \text{sign}\left((r\alpha t - 1 + \alpha)\left(\int_0^t \exp(-rs) s^{1/\alpha} ds\right)\exp(rt) + \alpha t^{(1+\alpha)/\alpha}\right) \tag{57}$$

The right hand side of this equation is an increasing function of t, so if $\dot{\omega}(0,x) \geq 0$, $\dot{\omega}(t,x) \geq 0$ for all t.

$$\dot{\omega}(0,x) = \lim_{t \to 0} \frac{(r\alpha t - (1+\alpha))\omega(t,x) + \beta q(x) + rU}{\alpha t} \tag{58}$$

L'Hôpital's rule implies

$$\dot{\omega}(0,x) = \frac{(r\alpha\omega(0,x) - (1+\alpha))\dot{\omega}(0,x)}{\alpha} \tag{59}$$

$$\dot{\omega}(0,x) = \frac{r\alpha\omega(0,x)}{1+2\alpha} > 0 \tag{60}$$

This completes the proof.

References

G. Becker (1962), "Investment in Human Capital: A Theoretical Analysis", *Journal of Political Economy*, 70, 9-49.

S. Bentolila and G. Bertola (1990), "Firing Costs and Labour Demand: How Bad is Euroesclerosis?", *Review of Economic Studies*, 57, 381-402.

M. Burda (1992), "A Note on Firing Costs and Severance Benefits in Equilibrium Unemployment", *Sacandinavian Journal of Economics*, 94, 479-489.

M. Despax and J. Rojot (1987), *Labour Law and Industrial Relations in France*, Kluwer Law and Taxation Publishers.

B. Hepple and S. Fredman (1992), *Labour Law and Industrial Relations in Great Britain*, Kluwer Law and Taxation Publishers.

H. A. Hopenhayn (1991), "Entry, Exit and Firm Dynamics in Long Run Equilibrium", *Econometrica*, 60, 1127-1150.

H. A. Hopenhayn (1992), "Exit, Selection and the Value of Firms", *Journal of Economic Dynamics and Control*, 10, 621-653.

H. A. Hopenhayn and W. García-Fontes, "Employment Creation and Destruction in the Spanish Economy", mimeo, UPF.

H. A. Hopenhayn and R. Rogerson (1993), "Job Turnover and Policy Evaluation: A General Equilibrium Analysis", *Journal of Political Economy*, 101, 915-938.

B. Jovanovic (1982), "Selection and the Evolution of Industry", *Econometrica*, 50, 649-670.

E. P. Lazear (1981), "Agency, Earnings Profiles, Productivity and Hours Restrictions", *American Economic Review*, 71, 606-620.

K. J. McLaughlin (1991), “A Theory of Quits and Layoffs with Efficient Turnover”, *Journal of Political Economy*, 99, 1-29.

S. P. Millard and D. T. Mortensen, (1995) “The Unemployment and Welfare Effects of labour Market Policy: A Comparison of the US and the UK”, mimeo, Northwestern University.

D. T. Mortensen and C. A. Pissarides, (1994) “Job Creation and Job Destruction in the Theory of Unemployment”, *Review of Economic Studies*, 61, 397-416.

J. Nash (1953), “Two-Person Cooperative Games”, *Econometrica*, 21, 128-140.

R. Topel (1991), “Specific Capital, Mobility and Wages: Wages Rise with Job Seniority”, *Journal of Political Economy*, 99, 145-176.