From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share*

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Abstract

The US economy has undergone several puzzling changes in recent decades. Large firms now account for a greater share of economic activity, new firms are being created at slower rates, and workers are receiving a smaller share of GDP. Changes in population growth provide a unified quantitative explanation. A decrease in population growth lowers firm entry rates, shifting the firm-age distribution towards older firms. Heterogeneity across firm-age groups combined with an aging firm distribution replicates the observed trends. Firm aging accounts for i) the concentration of employment in large firms, ii) and trends in average firm size and exit rates, key determinants of firm entry rates. Feedback effects from firm demographics generate two-thirds of the effect. Transitional dynamics within these feedback effects are key, accounting for half the total change. Firm aging increases the market share of large firms, which have lower labor shares, driving down the aggregate labor share.

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1 Introduction

Three long-term changes in the US economy have attracted a great deal of attention. First, economic activity is being concentrated in larger firms. For example, the fraction of workers employed by large firms has increased by 6 percentage points since 1978. Second, the entrepreneurship rate — the ratio of new firms to total firms — has nearly halved since the 1970s. Third, the share of GDP going to labor, once thought to be stable, has declined since 1975. What explains these changes?

Our analysis begins by highlighting the importance of changing firm demographics —an aging firm distribution combined with heterogeneity by firm age— in driving these aggregate trends. We document that the increase in employment concentration is entirely driven by changing firm demographics. There has been no change in employment concentration within firm-age categories. Nevertheless, aggregate concentration has increased because an aging firm distribution shifts weights towards older firms, which have higher employment concentration. We document that changing firm demographics can also account for changes in two related variables: average firm size and the aggregate firm exit rate. Conditional on age, these variables have changed little over time. However, because older firms are larger and exit at lower rates, an aging firm distribution has led to an increase in average firm size and a decline in the aggregate exit rate.

Because firm exit rates by age have changed little, the probability that a new firm survives to a particular age has also changed little. Therefore, the aging of firms is a result of the dramatic fall in the rate at which new firms are created. This decline in the entrepreneurship rate can be analyzed through the lens of a simple accounting identity. The firm entry rate equals the aggregate exit rate minus the growth in average firm size plus labor force growth,[1]

$$\underbrace{\lambda}_{\text{Entry Rate}} = \underbrace{\xi}_{\text{Exit Rate}} - \underbrace{\widehat{e}}_{\text{AFS Growth}} + \underbrace{\widehat{N}}_{\text{LF Growth}}. \quad (1)$$

[1]This identity comes from the definition of average firm size, $e \equiv N/M$, where N is the number of workers and M is the number of firms. It follows that the growth rate in the number of firms equals the growth rate in the number of workers minus the growth rate of average firm size, $\widehat{M} = \widehat{N} - \widehat{e}$. The growth in the number of firms also depends on firm entry and exit, $\widehat{M} = \lambda - \xi$. Combining these two equations leads to identity (1). We measure $\widehat{N}$ using labor force growth. Other measures of $\widehat{N}$ are discussed in Section 5.

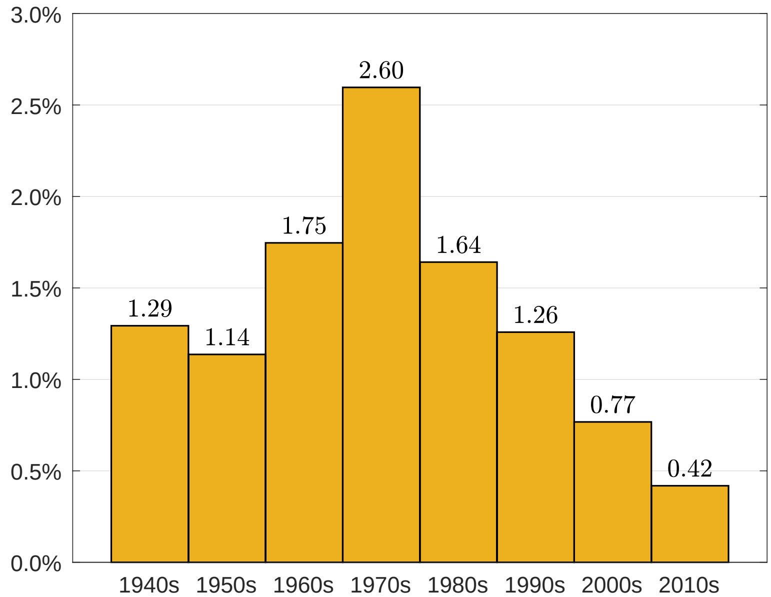


Figure 1: Civilian Labor Force Growth Rate

The exit rate and average firm size are constant in stationary equilibria of standard firm dynamics models. Therefore, changes in labor force growth are a natural candidate to explain changes in the firm entry rate. Holding the exit rate and average firm size constant, can a change in labor force growth explain the observed drop in firm entry rates? No. US labor force growth has declined, but not by enough. Figure 1 shows US civilian labor force growth rates by decade. Since the 1970s, labor force growth has declined by 2 percentage points (pp), which is only one-third of the 6pp decline in the entry rate. The remaining two-thirds is attributed to changes in the exit rate and changes in the growth rate of average firm size.

We show that changes in labor force growth feedback to changes in both the aggregate exit rate and average firm size. Consider an increase in labor force growth. The increase in labor supply must be met by a corresponding increase in labor demand. Incumbent firms are limited by scale, so they cannot absorb the entire increase in labor supply. The residual labor demand must therefore be absorbed by new firms. The increase in firm entry shifts the firm-age distribution towards younger firms, which have higher exit rates and lower size.

To be consistent with the data, the changes in labor force growth should change aggregate variables while maintaining stability of these variables by firm age. While this property holds along a balanced growth path, it is not clear that it carries over to transitions. This distinction is of interest because the growth in average firm size in the data is non-zero, indicating that the US economy is going through a transition. The theoretical challenge is to show that an evolving firm-age distribution along the transition path is consistent with stability of firm-level variables by age. We derive sufficient conditions for the existence of such an equilibrium in a general framework that incorporates standard models of perfect competition and models of imperfect competition featuring both constant and variable markups.

The transitional dynamics of firm entry depend on the entire history of past entry. Firm entry fills the gap between labor supply and incumbent labor demand. Therefore, entry depends on total labor demand by incumbents in each age group, which in turn is determined by past entry, survival probabilities and average size. We characterize this feedback with the *dynamic entry equation*, which relates current entry to the distributed lag of past entries. The dynamic nature of entry implies that changes in current entry affect future entry, through the firm-age distribution.

Can changes in labor force growth, combined with feedback from firm demographics, quantitatively generate the secular changes experienced by the US economy? What is the role of the feedback effect? What is the role of transitional dynamics, and therefore the importance of the baby boom? How do we expect entry rates and firm demographics to evolve from here on? We can answer these questions using the dynamic entry equation. To obtain labor demand by incumbents in each age group, we calibrate a stochastic process for firm employment that is consistent with key facts about firm size, firm growth and exit. We then feed the labor force series into the dynamic entry equation and iterate forward.

We find the decline in labor force growth can explain the majority of the observed decline in firm entry rates from 1978 to 2014. In addition, changes in labor force growth explain well two episodes in the data: the pre-1978 increase in the entry rate, and the large fluctuations in the entry rate around World War II. As in the data, the post-1978 decline in labor force growth generates a 2pp decline in the exit rate, a 6pp increase in employment concentration, an increase in average firm size and an aging of the firm distribution. To confirm that the results are not an artefact of our calibration strategy, we repeat the quantitative exercise using an alternative non-parametric approach. This approach obtains incumbent labor demand by directly assigning, or by imputing, average size and survival probabilities by firm age from available data. The results from the alternative approach are similar.

We decompose our results along two dimensions. The first decomposition uses identity (1) to break down the decline in firm entry into the direct effects of labor force growth and the feedback effects of firm demographics. The feedback effects are captured by the decline in the exit rate and the change in the growth rate of average firm size. As in the data, we find that the 6pp decline in the entry rate is accounted for by a 2pp drop in labor force growth, a 2pp drop in the exit rate, and a 2pp *increase* in the growth rate of average firm size. Therefore, the feedback effect of firm demographics accounts for two-thirds of the decline in the entry rate.

The second decomposition breaks down the decline in firm entry into long-run effects and transitional effects. In addition to the direct effect of population growth, the long-run effect depends on how changes in population growth affect the exit rate. We find that the long run elasticity of entry rates to population growth is in the order of 1.5 for the US economy. This implies that the 2pp drop in labor force growth generates a 3pp drop in entry rates, which corresponds to half of the 6pp decline in the data. Transitional dynamics account for the remaining half of the decline in entry rates. The transitional effects follow from the glut of firms born during a period of rising labor force growth. The transitional firm-age distribution is younger than the long-run distribution immediately after the rise in labor force growth, and is older than the long-run distribution in 2014 once the glut of firms have aged. This leads to a 1pp effect from transitional exit. In addition, the transitional firm-age distribution is getting younger in 1978, implying a negative average firm size growth rate, and it is getting older in 2014, implying a 2pp effect from growth in average firm size as compared to the long-run.

We explore the implications for future entry rates by feeding labor force projections to the year 2060 from the BLS and iterating the model forward. We find that despite a projected decline in labor force growth, future entry rates *bounce back* by 1pp. This can be understood using identity (1). The exit rate is projected to increase because the glut of firms born in the years of high labor force growth will have mostly died off and will have been replaced by younger firms, which have higher exit rates on average. The projected growth in average firm size hits zero because the economy is expected to converge to a balanced growth path. Both these forces more than counteract the effect of the future decline in labor force growth.

We next turn to the labor share. A recent set of papers document two facts about labor shares: (i) firm-level labor shares are negatively related to firm size and (ii) almost all of the decline in the aggregate labor share is due to reallocation from high to low labor share units, rather than a decline within labor share units.[2] Because older firms tend to be larger, an aging of the firm distribution corresponds to reallocation of value added towards larger firms. A decline in labor force growth lowers the aggregate labor share, by shifting the firm-age distribution towards older firms. This is consistent with the data: the decline in the corporate labor share in Karabarbounis and Neiman (2014) is coincident with the decline in labor force growth from 1980 onwards. Recent work by Koh, Santaeulalia-Llopis and Zheng (2018) extends the measurement of the corporate labor share back to 1947, and finds a hump-shaped pattern. This is the pattern predicted by labor force growth.

Declining population growth also has implications for job reallocation. As noted by Decker, Haltiwanger, Jarmin and Miranda (2014), older firms create, destroy, and reallocate jobs at lower rates. Therefore, firm aging also acts as a force that contributes to declining aggregate job reallocation rates. We find that, for the 1978 to 2014 period, firm aging induced by population growth accounts for 47 percent of the observed decline in job creation, 40 percent of the decline in job destruction, and 35 percent of the decline in job reallocation.[3]

We close by showing that changes in population growth are the primary driver of changes in labor force growth. We decompose labor force growth into three components: birth rates sixteen years prior, the growth in participation rates, and

[2] Hartman-Glaser, Lustig and Xiaolan (2019) documents this pattern by showing that the capital share has been increasing for the largest public firms in the US. Autor, Dorn, Katz, Patterson and Van Reenen (2017) document the same pattern using US Census Data. Kehrig and Vincent (2018) document the reallocation for manufacturing establishments.

[3] These numbers are larger than those found by previous empirical studies, such as Decker, Haltiwanger, Jarmin and Miranda (2014). The reason for this difference is that our theory allows us to fill the gaps in the firm-age distribution observed in the data, so we can perform an analysis with a finer age distribution and over a longer period of time.

a residual term that captures rates of migration, death and institutionalization. Birth rates sixteen years prior account for the bulk of the changes in labor force growth.

Related Literature. Our paper builds on a wealth of recent empirical evidence from seemingly disconnected strands of the literature. One strand of the literature has documented changes in entry rates and the age distribution of firms. Reedy and Strom (2012) document the decline in firm entry rates, while Pugsley and Şahin (2018), Decker, Haltiwanger, Jarmin and Miranda (2014), Hathaway and Litan (2014a), Gourio, Messer and Siemer (2015) and Davis and Haltiwanger (2014) document the pervasiveness of this decline across geographic areas and industries. Decker, Haltiwanger, Jarmin and Miranda (2014), Hathaway and Litan (2014b) and Pugsley and Şahin (2018) document the aging of the firm distribution and link it to declining firm entry. A different strand of the literature has documented trends in the aggregate labor share and the rise in concentration. Karabarbounis and Neiman (2014) find that the decline in the labor share is primarily a within-industry rather than a cross-industry phenomenon. Grullon, Larkin and Michaely (2017) document increased concentration across most U.S. industries, whereas Barkai (2017) and Autor, Dorn, Katz, Patterson and Van Reenen (2017) both document a positive correlation between industry concentration and the decline in the labor share. Our paper incorporates all of these empirical findings into one unified explanation.

We are not the first paper to propose the decline in labor force growth as an explanation for the decline in firm entry rates. Using lagged fertility rates as an instrument, Karahan, Pugsley and Şahin (2018) find that the entry rate is highly elastic to changes in labor supply across US states.[4] The authors then explore the role of labor force growth in the steady state of a Hopenhayn (1992a)-style model. There are two main differences between our papers. First, we aim to explain a broader set of facts, such as the increase in concentration and the decline of the labor share. Second, our study focuses on transitional dynamics, allowing

[4] Hathaway and Litan (2014c) also note a correlation between declining firm entry rates and population growth across geographic regions. Other explanations for the decline in entrepreneurship include the decline in corporate taxes (Neira and Singhania, 2019), the decline in interest rates, (Liu, Mian and Sufi, 2018; Chatterjee and Eyigungor, 2018), and skill-biased technical change (Salgado, 2018; Jiang and Sohail, 2017).

us to uncover how the history of past entry matters for current entry and firm demographics.

To the best of our knowledge, ours is the first paper that jointly explains the transitional dynamics of entrepreneurship, concentration, and the labor share.[5] One related, but distinct, explanation is that of the aging of the workforce (Liang, Wang and Lazear, 2018; Kopecky, 2017; Engbom, 2017).[6] We note that a decline in labor force growth is a different phenomenon than an aging workforce. Another observation that has gained considerable attention is that of the rise in markups (De Loecker and Eeckhout, 2017).[7] Our framework is consistent with rising markups provided older firms have higher markups.[8]

The rest of the paper is organized as follows. Section 2 presents the data. Section 3 presents the theoretical results. Section 4 presents the quantitative findings. Section 5 discusses the alternative quantitative approach, job reallocation, implications for productivity, drivers of labor force growth and alternative measures of labor supply. Section 6 concludes.

2 Motivating Facts

We obtain data on firms from the Business Dynamics Statistics (BDS) produced by the US Census Bureau. The BDS dataset covers the 1977 to 2014 period. It has

[5]Complementary explanations include an increase in the span of control (Aghion, Bergeaud, Boppart, Klenow and Li, 2019), and a decline in knowledge diffusion between frontier and laggard firms (Akcigit and Ates, forthcoming) . Explanations specific to the labor share decline include a slowdown in productivity (Grossman, Helpman, Oberfield and Sampson, 2017), an increase in firm-level volatility (Hartman-Glaser, Lustig and Xiaolan, 2019), the treatment of intangible capital (Koh, Santaeulalia-Llopis and Zheng, 2018), the decline in the relative price of capital (Karabarbounis and Neiman, 2014), capital accumulation (Piketty and Zucman, 2014), import competition and globalization (Elsby, Hobijn and Sahin, 2013), and corporate taxes (Kaymak and Schott, 2018).

[6]More broadly, population aging has been linked to slower growth in advanced economies; see Cooley and Henriksen (2018).

[7]Our framework shows that it is possible to generate an increase in concentration without decreasing competition. Rossi-Hansberg, Sarte and Trachter (2018) also show that increasing concentration at the aggregate level need not be due to declining competition. They present evidence that the positive trend observed in national product-market concentration becomes a negative trend when focusing on measures of local concentration.

[8]Contemporaneous work by Peters and Walsh (2019) find this to be the case in US data. They measure markups using the Longitudinal Business Database and derive steady state implications of declining population growth in a model of creative destruction.

near universal coverage of private sector firms with paid employees.

We start by looking at the time series evolution of concentration, average firm size and the aggregate exit rate in US data; see top panel of Figure 2. We measure concentration as the share of employment by firms with 250+ employees. Figure 2 shows that concentration in the US has increased from about 51 percent to 57 percent.[9] Average firm size in the US has increased steadily from about 20 employees to about 24 employees. The aggregate exit rate has declined steadily from about 9.5 percent to about 7.5 percent. The bottom panel of Figure 2 shows the time series of concentration, average firm size and exit rates broken down by firm age. None of the aggregate changes have occurred within firm-age bins. For example, a typical five-year-old firm has the same size in 1980 and 2014, with no discernible trend. The same pattern holds for concentration and exit rates: conditional on age, concentration and exit rates do not exhibit a trend over the 1977-2014 time period. It follows that the aggregate trends in concentration, average firm size and exit rates are not being driven by changes in the corresponding variables within firm-age categories.

The bottom panel of Figure 2 also shows that there are significant differences in levels across age groups. Concentration and average firm size increase with age. Firm exit rates decrease with age. These patterns suggest that changes in the age composition of firms drive the aggregate trend in each variable. In order to investigate this formally we run the following regression,

$$y_{ajt} = \beta_0 + \beta_y\, year + \sum_a \beta_a\, age + \sum_j \beta_j\, sector + \sum_a \sum_j \beta_{aj}(age \times sector) + \varepsilon_{ajt},$$

where y_{ajt} is the employment share of firms with 250+ employees, log average firm size or firm exit rates. To rule out the possibility that our concentration results are being driven by the size threshold of 250 employees, we also run the regression for size thresholds of 500, 1000, 2500, 5000 and 10,000 employees. We start with a specification that only regresses the dependent variable on *year* and an intercept term. The coefficient on *year* captures the aggregate trend in the dependent variable. We then add age controls and see how the *year* coefficient

[9] The increase in concentration is robust to the firm size cutoff. For size cutoffs of 5, 10, 20, 50, 100, 250, 500, 1000, 2500, 5000, and 10,000 employees, the share of employment increased by 1.6, 3.1, 4.3, 5.4, 6.0, 5.7, 5.1, 4.6, 3.9, 3.1, and 2.4 percentage points, respectively.

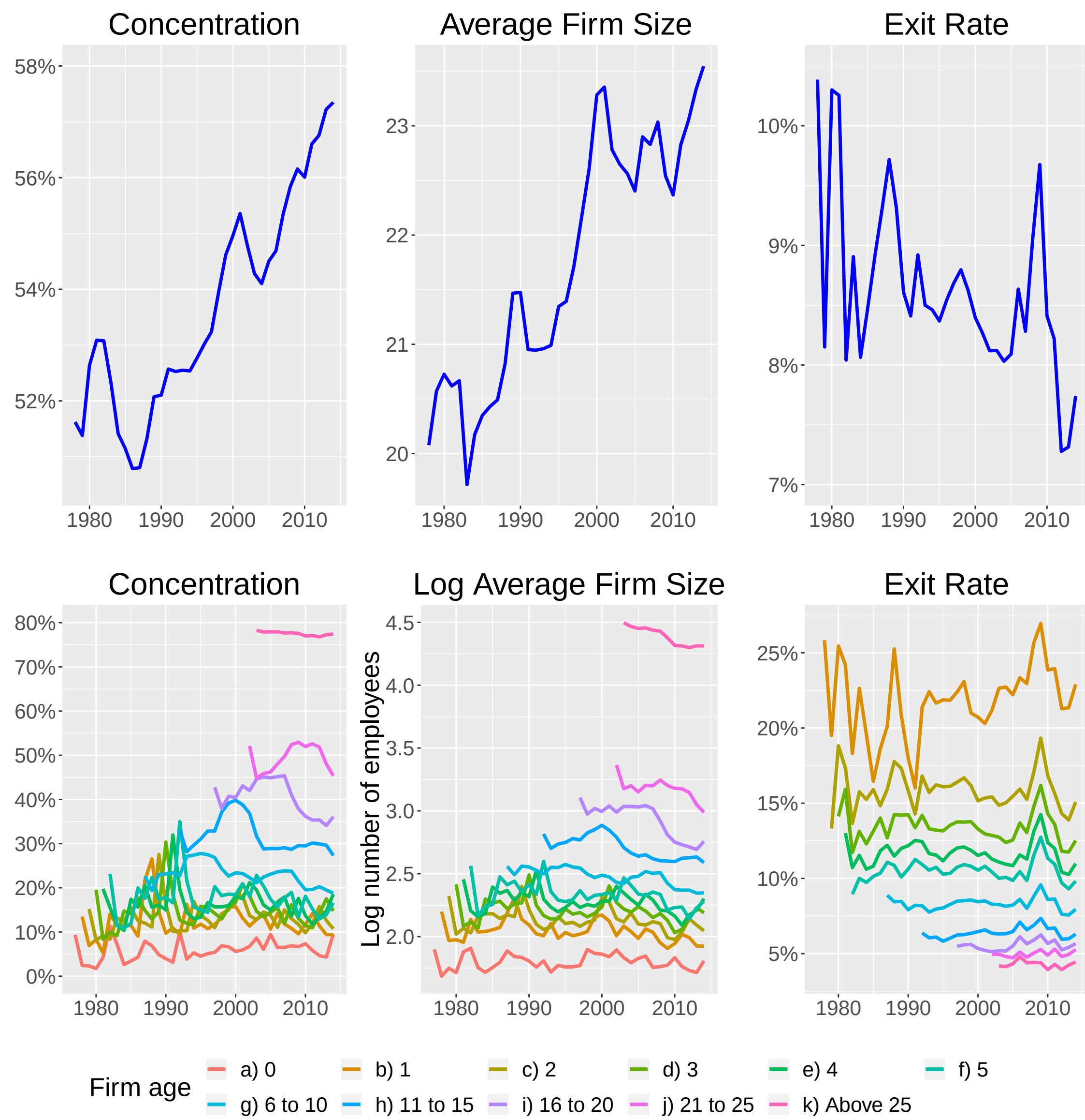


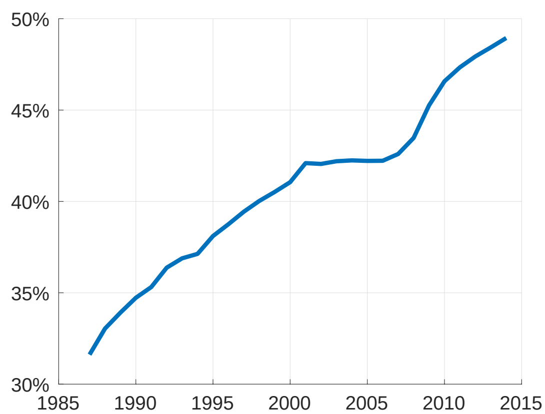
Figure 2

Source. US Business Dynamics Statistics.
Notes. Concentration is the share of employment in firms with 250+ employees. Concentration within an age category is share of employment in firms with 250+ employees within the age category divided by total employment in the age category. The *Above 25* age category includes firms labeled 26+ and Left Censored firms in the Business Dynamics Statistics. Average firm size is number of workers per firm.

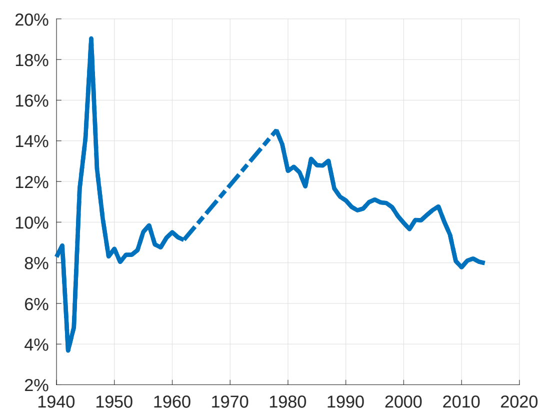
changes. For the average firm size and firm exit rate regressions, we add further controls for sector and age-sector interaction effects in successive specifications.[10]

[10]To protect the identity of firms, the Business Dynamics Statistics do not report data on share

The regression results confirm that changes in the age composition drive the aggregate trends; see Tables B-1 to B-3 in Appendix B. Without controls, the average trend across age groups and sectors in each variable is statistically significant and non-zero. Once we control for age, however, the trend disappears or reverses sign. The inclusion of controls for sector and age-sector interactions has no further effect on the trend. The coefficients on the age controls exhibit the same patterns as Figure 2: they increase with age for average firm size and concentration, and decrease with age for exit rates. The results on concentration do not depend on the size threshold, indicating that concentration within age bins has not increased even for the largest firms in BDS data.



(a) Share of Firms Age 11+



(b) Entry Rate

Figure 3

Sources. (a) BDS. (b) Entry rate 1940-1962: Survey of Current Business. Entry rate 1978-2014: BDS. *Notes.* The entry rate from 1963 to 1977 is linearly interpolated.

Figure 3a presents direct evidence that US firms are aging. The figure shows that the share of firms aged 11+ has risen steadily from 32 percent in 1986 to 48 percent in 2014. Figure 3b shows the contemporaneous decline in entry rates. The entry rate series can be extended back to 1940. Two episodes stand out in the early period of the entry rate series. First, the entry rate displayed large fluctuations around World War II. Second, the entry rate displayed an apparent increase before 1978.[11]

of employment by firm size, age and sector. Therefore, we cannot include controls for sector and age-sector interactions in the concentration regression.

[11]The entry rate from 1940 to 1962 comes from the now discontinued Survey of Current Business. The entry rate from 1963 to 1977 is linearly interpolated. The apparent increase in that

3 Theory

This section considers a class of models which share the property that equilibrium allocations are time-invariant, when conditioned on firm age, even in the presence of nonstationary changes in population. We first provide the basic structure that is common to this class of models, and then give some specific examples. For clarity of exposition, we have chosen to keep the basic structure as simple as possible. Our goal is to provide a minimal framework to help organize and interpret the empirical evidence discussed earlier.

There is a fixed endowment of a labor N_t, which is inelastically supplied and also the numeraire. Firms are confronted with an aggregate state z (e.g. price index) and an idiosyncratic state s, which for simplicity we call quality. The aggregate state z is determined as part of the equilibrium. The idiosyncratic state s follows a Markov process with conditional distribution $F(s_{t+1}|s_t)$, which we assume is continuous and nondecreasing. Let $\pi(s,z)$ denote the profits of a firm of quality s when the aggregate state is z, with $n(s,z)$ denoting employment. We assume both functions are increasing in s and z. Firms have a common discount factor β. Note that, for fixed z, the Markov process for a firm's state s_{it} and the function $n(s,z)$ determine the evolution of firm employment. In turn, this Markov process and the profit function $\pi(s,z)$ play a key role in determining the properties of firm survival, as explained below. [12]

The technology for entry of a new firm is as follows. Upon paying a cost of entry of c_e units of labor, entrants draw their initial productivity from a fixed distribution G. The productivity draws are independent across entrants and time. This assumption implies that potential entrants are ex-ante identical and get differentiated ex-post as a result of their initial draws and the stochastic evolution. While admittedly extreme, it captures in a stylized way the large amount of uncertainty faced by potential entrants. As discussed below, this assumption implies a perfectly elastic supply of potential entrants and will play a key role in our

period is consistent with the increase in the entry rate for *establishments* documented by Karahan, Pugsley and Şahin (2018).

[12] It is worth noting that the basic framework can easily be extended in several ways. We can include (1) multiple factors of production, with the assumption of some common aggregator, (2) R&D can be introduced assuming the Markov process for the firm's state is affected by the resources (e.g. labor) employed in R&D, (3) the stochastic process for s_{it} need not be Markov, and thus firm-age effects or learning as in Jovanovic (1982) can be easily introduced.

analysis.

3.1 Examples

Our formulation is general and can encompass models of perfect and imperfect competition.

Perfect competition. Firms produce a homogeneous good with labor input n with production technology $q(s,n)$, where s can be interpreted as a productivity shock. Assume q is increasing in s, supermodular and strictly concave. In addition there is a fixed cost of production c_f which captures labor overhead. The model is the standard entry and exit model considered in the literature based on Hopenhayn (1992a). Let z_t be the price of the output good in units of labor, the numeraire. Profits are given by

$$\pi(s,z) = \max_n zq(s,n) - n - c_f$$

and employment $n(s,z)$ is the unique maximizer. Given the above assumptions it follows immediately that both $\pi(s,z)$ and $n(s,z)$ are increasing in s and z.

Monopolistic competition with constant elasticity. Each firm i produces a differentiated good with a linear production function, $q(i) = s(i)n(i)$ and a fixed cost c_f in units of labor. The representative consumer has preferences over intermediate goods given by the aggregator

$$U = \left(\int c(i)^{\eta} \, di \right)^{1/\eta}$$

where $0 < \eta < 1$, and $c(i)$ denotes total consumption of firm i's output. First order conditions for the choice of $c(i)$ are given by

$$U^{1-\eta} c(i)^{\eta-1} = \theta p(i),$$

where θ is the multiplier of the budget constraint of the consumer.[13] Revenues per consumer

$$\begin{aligned} p(i)\, c(i) &= U^{1-\eta}\theta^{-1} c(i)^{\eta} \\ &= U^{1-\eta}\theta^{-1} \left(s(i)\, n(i) / N\right)^{\eta} \end{aligned}$$

Given a population of consumers N and letting $z = N^{1-\eta}U^{1-\eta}\theta^{-1}$, total firm revenues are $R(s,n,z) = z(sn)^{\eta}$. Profits are given by

$$\pi(s,z) = \max_{n} R(s,n,z) - n - c_f$$

and employment $n(s,z)$ is the unique maximizer. It follows immediately that both the profit and employment functions are increasing in s and z.

Monopolistic competition with variable markups. There is a continuum of firms each producing a different variety of a final good with quality $s(i)$. Preferences of the representative consumer are given by the CES aggregator:

$$U = \left(\int s(i)\, c(i)^{(\sigma-1)/\sigma} di \right)^{\sigma/(\sigma-1)}$$

where $\sigma > 1$ is the elasticity of substitution between varieties and $c(i)$ denotes consumption per capita of good i. Firms produce with linear technology $q(i) = n(i)$. Suppose the marginal cost of producing an existing product by a firm of quality s is one. An outside imitator can make this product at marginal cost $1 < p(s) < \frac{\sigma}{\sigma-1}$, as in the literature based on Grossman and Helpman (1991). All costs are expressed in units of labor. As a consequence of limit pricing by potential imitators, the price of a good of quality s will thus be equal to $p(s)$. If $p(s)$ is an increasing function, markups will increase in s. Order the preferences of the representative consumer by product quality and let $\mu(s)$ denote the measure

[13] An alternative equivalent formulation is that U represents a final good produced by perfectly competitive firms with the production function given by the aggregator above. In that case, θ^{-1} is the price of the final good.

of firm qualities, we have

$$U=\left(\int sc\left(s\right)^{(\sigma-1)/\sigma}d\mu\left(s\right)\right)^{\sigma/(\sigma-1)}$$

This gives individual demand functions of the form

$$c\left(s\right) \;=\; U\theta^{-\sigma}\left(s/p\left(s\right)\right)^{\sigma},$$

where θ is the multiplier of the budget constraint of the consumer.

Profits of a firm of quality s are

$$\pi\left(s,z\right)=z\left(s/p\left(s\right)\right)^{\sigma}\left(p\left(s\right)-1\right)-c_{f}$$

where $z=NU\theta^{-\sigma}$ and c_f is overhead labor. Likewise, total output and employment $n\left(s,z\right)$ of firm of quality s equals $Nc\left(s\right)$, so

$$n\left(s,z\right) \;=\; z\left(s/p\left(s\right)\right)^{\sigma}.$$

Assuming $s/p\left(s\right)$ is increasing in s, it follows immediately that both profits and employment are increasing in s and z. Note also that with these assumptions, higher quality firms are larger, i.e. employ more workers, and have higher markups.[14]

3.2 Equilibrium

Given a deterministic path for the aggregate state $\mathbf{z}_t=\{z_\tau\}_{\tau\geq t}$, the present discounted value of a firm is given by the Bellman equation:

$$v\left(s,\mathbf{z}_t\right)=\max\left\{0,\pi\left(s,z_t\right)+\beta Ev\left(s',\mathbf{z}_{t+1}|s\right)\right\}.$$

The value of exit is normalized to zero, while the right-hand side under the maximization is the continuation value for the firm. It is easy to show that, when

[14] In equilibrium, there is a unique constant level of $z=z^e$ that satisfies the zero net value condition for entrants, as discussed in Section 3.2, where we construct the unique allocation that supports this constant value z^e as an equilibrium.

nonzero, this value is increasing in s and z_t. Let

$$s_t^* = \inf \left\{ s | \pi \left(s, z_t \right) + \beta E v \left(s', \mathbf{z}_{t+1} | s \right) > 0 \right\}. \tag{2}$$

A firm is shut down iff $s \leq s_t^*$.[15]

Prior to entry, the expected value of an entrant net of the entry cost is

$$v^e \left(\mathbf{z}_t \right) = \int v \left(s, \mathbf{z}_t \right) dG \left(s \right) - c_e. \tag{3}$$

Let μ_t denote the measure of firms operating at time t, where for a fixed set A of firm types, $\mu_t \left(A \right)$ measures the magnitude of firms that at time t have $s_{it} \in A$. Given an initial measure μ_0, the exit thresholds s_t^* together with mass of entrants m_t determine uniquely the sequence of measures $\{\mu_t\}$ recursively as follows. For any set of productivities A, define

$$\mu_{t+1} \left(A \right) = m_{t+1} \left(\int_{s \epsilon A, s \geq s_{t+1}^*} dG \left(s \right) \right) + \int \int_{s \epsilon A, s \geq s_{t+1}^*} dF \left(s | x \right) d\mu_t \left(x \right) \tag{4}$$

The first term in the right-hand side corresponds to entrants, excluding those that exit immediately, while the second term corresponds to incumbents after the realization of new productivities, excluding those that exit.

Let $M_t = \int d\mu_t \left(s \right)$ denote the total mass of firms. The resource constraint requires that

$$\int n \left(s, z_t \right) d\mu_t \left(s \right) + \int c_f d\mu_t \left(s \right) + m_t c_e = N_t. \tag{5}$$

The first term is productive labor demand, the second is overhead labor and the third is labor utilized for creation of entrants. The right-hand side represents total labor, which is inelastically supplied.

An equilibrium for a given sequence $\{N_t\}$ and given initial measure μ_0 is given by shutdown thresholds $\{s_t^*\}$, mass of entrants $\{m_t\}$, measures of firms $\{\mu_t\}$ and aggregate states $\mathbf{z}_t = \{z_t\}$ such that:

1. Exit: Shutdown thresholds are given by equation (2);

[15] Under some regularity conditions (Hopenhayn, 1992b, see) it can be guaranteed that such a unique finite threshold s_t^* exists. These conditions amount to profits being negative for sufficiently low quality s, and that shocks are persistent.

2. Entry: No rents for entrants, $v^e(\mathbf{z}_t) \leq 0$ and $v^e(\mathbf{z}_t) m_t = 0$;

3. Resource constraint (5) holds.

4. Law of motion: The sequence μ_t is generated recursively by equation (4) given the initial measure μ_0.

We focus on equilibria with strictly positive entry, which is the relevant case in reality. Along the lines of Hopenhayn (1992a), it can be shown that a stationary equilibrium exists and is unique when labor N_t grows at a constant rate. Here we generalize this result to the case where labor is growing at non-constant rates. In particular, we provide conditions for the existence and uniqueness of a *constant aggregate state equilibrium*, $z_t = z^*$ for all t.[16] Under the above assumptions, it can be shown that there is a unique value z^* for which the expected value of an entrant is zero, $v^e(z^*) = 0$. This value of z^* corresponds to the aggregate state in the stationary equilibrium in Hopenhayn (1992a). The intuition behind the equilibrium construction is as follows. At this value z^* there is a perfectly elastic supply of potential entrants. Adjustments in this extensive margin guarantee that the market clearing condition (5) holds in every period, provided that the implied number of entrants is never negative. In what follows, we develop the existence argument in detail.[17] Readers who want to skip the details can jump to Corollaries 1 and 2, which summarize the key equilibrium properties used in our quantitative analysis.

For existence, we need to show that the equilibrium conditions hold in every period. Let z^* be such that the entry condition holds, $v^e(z^*) = 0$. Let $s_t^* = s^*$ be

[16] A constant aggregate state equilibrium features interest rates that are time invariant. This can be rationalized by assuming that all agents are risk neutral, or alternatively by considering a small open economy. Our analysis should then be considered an approximation to equilibrium behavior in a model with variable interest rates. In our numerical calculations, we verify that the fluctuations in the implied path for aggregate consumption in such an economy are small, and under standard levels of curvature imply small changes in interest rates. If there is an aggregate trend in productivity growth, the aggregate state would grow at a rate proportional to productivity trend growth.

[17] The key assumption in the following construct is a perfectly elastic supply of new firms at the cost of entry c_e. This follows from the assumption that all firms draw from the same distribution $G(s)$ regardless of the number of entrants, so that there is no ex-ante heterogeneity. As usual in models where there is a linear margin for adjustment, it is changes along this margin that guarantee market clearing while keeping constant other margins. An analogue is the familiar case of a perfectly competitive industry with perfectly elastic supply at minimum average cost, where all changes in demand would be met by changes in the number of entrants at this constant price.

the corresponding shutdown threshold, so that the exit condition holds. Given μ_0, we construct the sequence μ_t recursively such that the law of motion holds.

It remains to verify that the resource constraint holds. Let S_a denote the probability that an entrant survives at least a periods, i.e. that the state $s_{i\tau} \geq s^*$ for ages τ from 0 to a. Let $\tilde{\mu}_a$ denote the cross-sectional probability distribution of productivities for firms in the cohort of age a. These can be obtained recursively as follows:

1. Let $S_0 = (1 - G(s^*))$. Let $\tilde{\mu}_0(ds) = G(ds)/S_0$ denote the distribution of entrant productivity draws conditional on $s \geq s^*$.

2. Let $S_a = S_{a-1} \int P(s_a \geq s^* | s_{a-1})\, d\tilde{\mu}_{a-1}(s_{a-1})$, where the term under the integral is the probability that a firm in cohort $a-1$ is not shut down in the next period, and let

$$\tilde{\mu}_a(ds) = \frac{\int P(ds_a | s_{a-1})\, d\tilde{\mu}_{a-1}(s_{a-1})}{S_a / S_{a-1}}.$$

Let $\tilde{e}_a = \int (n(s, z^*) + c_f(s)) d\tilde{\mu}_a$ denote the average employment of a firm in the age n cohort. Let E_{ta} denote total employment by that cohort at time t. In addition to average employment $\tilde{e}_a$, total employment E_{ta} depends on the original mass of entrants in that cohort and the survival rate,

$$E_{ta} = m_{t-a} S_a \tilde{e}_a.$$

Total employment by incumbents (i.e. excluding new entrants) at time t is the sum of employment by cohorts with age greater than one, $E_t^I = \sum_1^t E_{ta}$. On adding E_t^I and total employment by entrants $m_t(S_0 \tilde{e}_0 + c_e)$, we recover the resource constraint:

$$N_t = m_t(S_0 \tilde{e}_0 + c_e) + E_t^I. \tag{6}$$

Given that z^* is constant, S_a and $\tilde{e}_a$ are known at time t. Because m_{t-a}, and therefore E_t^I, are also known at time t, the only unknown in the above equation is m_t. It follows that equation (6) implicitly determines m_t such that the resource constraint holds. If m_t is strictly positive, all equilibrium conditions hold and the existence argument is complete. This occurs provided that $E_t^I < N_t$ in every period t. The following proposition provides sufficient conditions for strictly positive entry.

Proposition 1 (Constant Aggregate State Equilibrium)**.** *Suppose that N_t is a non-decreasing sequence and $S_a\tilde{e}_a$ is non-increasing. Then the aggregate state and the exit threshold are constant in the unique equilibrium, $z_t = z^*$ and $s_t^* = s^*$.*

The intuition is as follows. Because N_t is a nondecreasing sequence, a sufficient condition for $E^I_{t+1} < N_{t+1}$, which guarantees strictly positive entry in period $t+1$, is that $E^I_{t+1} < N_t$. Note that

$$N_t = m_t S_0 \tilde{e}_0 + m_{t-1} S_1 \tilde{e}_1 + ... + m_0 S_t \tilde{e}_t \ + \ m_t c_e$$
$$E^I_{t+1} = m_t S_1 \tilde{e}_1 + m_{t-1} S_2 \tilde{e}_2 + ... + m_0 S_{t+1} \tilde{e}_{t+1}.$$

Therefore E^I_{t+1} is the inner product of the same vector of the mass of entrants as N_t, with a forward shift in the corresponding terms $S_a\tilde{e}_a$ and without the entry cost term $m_t c_e$. A sufficient condition for $N_t - E^I_{t+1} > 0$ every period is that $S_a\tilde{e}_a$ decreases with a. For a given cohort, this condition is equivalent to saying that the total employment of the cohort is decreasing in age. In the data, survival rates are decreasing in a but average size of a cohort, when properly calibrated, is increasing. Therefore the sufficient condition holds when shutdown rates are sufficiently high to offset the growth in average size.[18] The necessary and sufficient condition for a constant aggregate state equilibrium is that entry is strictly positive, which is verified in our quantitative exercise. Now we discuss various properties of the equilibrium.

Corollary 1 (Time Invariance)**.** *Exit rates by age, average firm size by age, and size distributions by age are time invariant in a constant aggregate state equilibrium.*

This Corollary follows because a constant aggregate state implies that firm exit decisions and optimal scale of operation do not change over time. The law of large numbers applies to a cohort at each age, and therefore the firm demographic variables, S_a and $\tilde{e}_a$, and the size distribution by age are time invariant. It follows

[18]In the model, this property is easy to verify given the stochastic process for the idiosyncratic shocks s_{it} and the shutdown threshold s^*. Models that assume permanent productivity shocks and exogenous exit trivially satisfy this condition. The same holds true for the models where productivity shocks are redrawn with some probability from the same distribution as entrants, e.g. Mortensen and Pissarides (1994).

that the constant aggregate state equilibrium qualitatively generates the constancy by age of exit rates, average firm size and employment concentration as observed in the data. As a consequence, the Corollary implies that changes in aggregate variables, which weighted averages involving firm demographic variables S_a and $\tilde{e}_a$, will be entirely due to changes in weights.

Because employment by incumbents E_t^I depends on S_a and $\tilde{e}_a$, the mass of entrants in equation (6) depends on firm demographics. With strictly positive entry, we can solve for m_t to obtain the following result.

Corollary 2 (Dynamic Entry Equation)**.** *The mass of entrants in equilibrium is given by*

$$m_t = \frac{N_t - \sum_{a=1}^{\infty} m_{t-a} S_a \tilde{e}_a}{S_0 \tilde{e}_0 + c_e}. \tag{7}$$

Because S_a and $\tilde{e}_a$ are time invariant in equilibrium, the dynamic entry equation implies that the mass of entrants m_t is linear in N_t. It follows that, in equilibrium, changes in N_t are accommodated along the extensive margin by changes in entry. The dynamic entry equation also shows that entry in the constant aggregate state equilibrium is history dependent: current entry m_t depends on past entry m_{t-a}. Given N_t, higher entry in the past lowers current entry by increasing the mass of incumbent firms m_{t-a}. History dependence implies that a one-time shock to firm entry today will have persistent effects on future entry through its effect on the mass of incumbents m_{t-a} in future periods.

3.3 The Turnover of Firms

In this section we examine the determinants of aggregate rates of entry and exit. In particular, we highlight the role of firm demographics, i.e. the age distribution of firms, in determining aggregate entry and exit rates. We show that changes in firm demographics have important feedback effects on the entry rate along transitions, i.e. when population N_t is growing at non-constant rates.

The mass of aggregate exit at time t, denoted X_t, is the sum of exit masses of different age cohorts. Exit of firms of age a equals the difference in survival rates $S_{a-1} - S_a$ multiplied by the size of the cohort at entry, m_{t-a}. We follow here the convention that the age at which a firm is shut down corresponds to the age at which the firm was last productive. The model allows for entrants to exit immediately without producing, so the mass of immediate exits $m_t(1 - S_0)$ are excluded from aggregate exit. It follows that the mass of aggregate exit is given by

$$X_t = \sum_{a=1}^{t} m_{t-a} \left(S_{a-1} - S_a \right).$$

The number of firms at $t - 1$ is given by

$$M_{t-1} = \sum_{a=1}^{t} m_{t-a} S_{a-1}.$$

Let $\omega_{ta} \equiv m_{t-a} S_a / M_t$ denote the share of firms of age a in the total mass of firms at time t. The hazard rate of exit for a firm of age $a - 1$ is $(S_{a-1} - S_a)/S_{a-1}$. The aggregate exit rate $\xi_t \equiv X_t / M_{t-1}$ can be expressed as the weighted average of hazard rates of exit of different cohorts

$$\xi_t = \sum_{a=1}^{t} \omega_{t-1,a-1} \left(\frac{S_{a-1} - S_a}{S_{a-1}} \right). \tag{8}$$

The hazard rates of exit are fixed by the Time Invariance Corollary. Therefore, the aggregate exit rate is only a function of the age distribution of firms, which in turn is determined by past entry. This formula highlights the role of firm demographics in determining the aggregate exit rate. Because the hazard rates are different across firm ages, a change in the age distribution of firms affects the aggregate exit rate. The exception, of course, is when hazard rates are the same for all cohorts. In that case, the exit rate is independent of the age distribution.

Now consider entry rates. Following the convention about exit, we define $m_t S_0$ as the measure of entry.[19] Let $e_t = N_t / M_t$ denote average employment. The rate of growth in the number of firms is

$$\frac{M_t}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t}. \tag{9}$$

Letting $\overline{S}_t$ denote the average survival rate from $t - 1$ to t. The mass of firms M_t can be decomposed into the mass of surviving incumbents plus the mass of

[19]If we had we assumed that all entrants must produce for at least one period, then $S_0 = 1$ and m_t would be measured entry.

entrants,

$$M_t = \overline{S}_t M_{t-1} + m_t S_0.$$

Solving for M_t in (9) and substituting in the above equation gives the following expression for the entry rate

$$\lambda_t \equiv \frac{m_t S_0}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t} - \overline{S}_t, \tag{10}$$

which is the discrete-time version of identity (1).

Long-Run vs. Adjustment Path. Suppose we are on a balanced growth path and population grows at a constant rate g. Average employment e_t is constant along this path. The cohort entry weights m_{t-a} decay as a function of age at the rate $1+g$, so $m_{t-a} = (1+g)^{-a} m_t$. The aggregate exit rate along the balanced growth path, denoted ξ^B, follows from (8)

$$\xi^B = \frac{\sum_{a=1}^{\infty} (1+g)^{-a} (S_{a-1} - S_a)}{\sum_{a=0}^{\infty} (1+g)^{-a} S_a}, \tag{11}$$

which is independent of t. The rate of population growth g and the long run exit rate ξ_t^B are intimately related to the hazard rate profile. Letting $h_a = (S_{a-1} - S_a) / S_{a-1}$ denote the probability of exit at age a, equation (11) can be rewritten as:

$$\xi^B = \frac{\sum_{a=1}^{\infty} (1+g)^{-a} S_a h_a}{\sum_{a=0}^{\infty} (1+g)^{-a} S_a}, \tag{12}$$

This equation shows that the aggregate exit rate is an average of the exit rates of different cohorts, weighted by the corresponding share of surviving firms in each cohort $(1+g)^{-a} S_a$. It is easy to see that, for higher values of g, the weights decrease faster with age a. Empirically, younger firms exit at higher rates. Therefore higher population growth, which shifts weight towards younger firms, is associated with higher long-run aggregate exit rates.

Proposition 2 (Long-run Multiplier Effect)**.** *Assume hazard rates of exits h_a are decreasing in cohort age a. Then the long-run aggregate exit rate ξ^B is increasing in population growth g.*

Because average firm size e_t is constant in a balanced growth path and $\overline{S}_t =$

$1 - \xi_t$, the entry rate in (10) along the balanced growth path, denoted λ^B, is also independent of t.[20] We have

$$\lambda^B = g + \xi^B.$$

The entry rate equals the sum of the population growth rate and the exit rate. The intuition is simple. Entrants must replace the exiting firms. In addition, because average employment is constant, the total mass of firms needs to grow at the rate of population growth, g, to clear the labor market. Therefore, there must be enough entry to also create this extra employment. When hazards rates of exit are decreasing in age, as occurs in practice, a change Δg in the rate of population growth leads to a change in the long-run aggregate exit rate $\Delta \xi^B$ of the same sign, as implied by Proposition 2. Therefore, the effect of population growth on the long-run rate of entry is amplified, $\Delta \lambda^B > \Delta g$. We refer to this as the long run multiplier effect. We find that it is in the order of 1.5 for the US economy.

More generally, when labor grows at non-constant rates and we are in a constant aggregate state equilibrium, changes in firm demographics have feedback effects on entry. The aggregate exit rate in that case is also a weighted average of cohort exit rates h_a. The weights, however, depend on the age distribution of firms and thus on the history of past entry. Because conditional exit rates are decreasing in age, a larger share of young firms is associated with a higher aggregate exit rate, and consequently with higher entry. In addition, changes in average employment further impact the entry rate. An initial rise in entry rates increases the share of younger firms which tend to be smaller. This lowers average firm size and, from equation (10), further increases the rate of entry. Thus, a rise in population growth leads to an increase in entry rates over and above those needed to accommodate the increase in the labor supply. This multiplier effect operates similarly in the opposite direction when population growth decreases.

The effect of transitional dynamics can be illustrated by considering the impact of a period of rising population growth, as exhibited by the US economy after the baby boom. Compared to the balanced growth path with constant population growth, rising population growth over time shifts the firm-age distribution toward

[20] The same holds in a model where productivity shocks are fully persistent or randomly redrawn from the same distribution as the one faced by entrants (as in Mortensen and Pissarides (1994)), average firm size is constant so the above formula applies. In particular this means that the rate of entry is independent of history and only depends on current population growth. If exit rates are not age dependent, the same will also be true for exit.

younger cohorts. This results in aggregate exit rates greater than, and average size lower than, the balanced growth path. Even in the hypothetical case that the rate of population growth is constant at its peak after the initial rise in the 1970s, the US economy should have seen a decrease in entry rates, as the aggregate exit rate and average firm size converge to their corresponding long-run values. Section 4.2 explores the relative importance of long-run vs transitional effects by decomposing changes in US entry rates into these effects.

4 Quantitative Analysis

The theory section provided conditions under which changes in labor force growth maintain constancy of firm demographics variables by age, while allowing for aggregate variables to evolve due to changes in the age-distribution. In this section, we explore the quantitative implications of the theory. We address the following questions. Can changes in labor force growth, combined with feedback from firm demographics, quantitatively generate the secular changes experienced by the US economy? What is the role of the feedback effect? What is the role of transitional dynamics, and therefore the importance of the baby boom? How do we expect entry rates and firm demographics to evolve from here on?

These quantitative questions can be answered using the dynamic entry equation (7). This equation determines the evolution of the firm-age distribution given an exogenous labor force series, an initial age distribution, and firm demographics variables for all ages. There is reliable data on labor force growth. However, there is limited data on the initial age distribution and firm demographics variables. Specifically, the Census does not assign an age to firms born before 1977. This implies that (i) the 1977 firm-age distribution is unknown, and that (ii) firm demographics variables for firms born before 1977 are also unknown.

Both these issues can be overcome by calibrating a stochastic process for employment. The employment process consists of the distribution of entrant employment, the evolution of employment over time and an exit rule. Therefore, the employment process implies values for the firm demographic variables for all ages. To obtain the 1977 firm-age distribution, we make use of historical labor force data which goes back to 1940. We assume the US economy was on a balanced growth path in 1940, so the 1940 firm-age distribution corresponds to the stationary distribution of the employment process. We then feed labor force growth and iterate the dynamic entry equation forward to obtain the firm-age distribution in 1977. Doing so allows us to obtain the 1977 age distribution that is consistent with historical labor force growth.[21]

The calibration we present is consistent with any economy that generates the same employment process and falls within the theoretical framework laid out in section 3. We present the calibrated parameters for two such economies, the perfect competition economy and the monopolistic competition with constant markup economy.[22] These two economies are isomorphic, once the parameters are appropriately reinterpreted.

The model period is set to one year. The time discount factor β is set to 0.96. The steady-state labor force growth rate g is set to the standard value of one percent. The aggregate state z^* and the idiosyncratic state s enter the profit function multiplicatively, which leads to an identification problem. We normalize the aggregate state z^* to one to get around this identification problem, as in Hopenhayn and Rogerson (1993). A parameter α captures the curvature in the revenue function. This parameter is set to the standard value of 0.64. In the perfect competition economy, the parameter α represents the degree of decreasing returns to scale in the production function of a firm, with $q(n, s)$ equal to sn^{α} , which can be interpreted as the managerial span of control. In the economies featuring monopolistic competition with constant elasticity and variable markups this parameter maps to the elasticity parameter η. The value of η equal to 0.64 implies that the elasticity of substitution, $1/(1-\eta)$, is close to its standard value of 3.

The idiosyncratic state s follows an AR(1) process,

$$\log(s_{t+1}) = \mu_s + \rho \log(s_t) + \varepsilon_{t+1}; \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2) \tag{13}$$

with ρ as the persistence, μ_s as the drift and σ_ε^2 as the variance of shocks. We allow overhead labor to increase monotonically with firm productivity, $c_f(s) = c_{fa} + c_{fb}s^{\frac{1}{1-\alpha}}$, to capture the intuitive idea that overhead labor increases with the number of production workers in the firm. The distribution of entrant produc-

[21]In Section 5 we report an alternative quantitative approach that does not rely on specifying a stochastic process for employment but rather directly assigns or imputes firm demographic variables from available data.

[22]Appendix D discusses how to map this calibration to the variable markup economy.

tivities G is lognormal with mean μ_G and variance σ_G^2. The total employment process at the firm level is a composite process that depends on the productive employment process, the constant exit threshold s^* and the process followed by overhead labor $c_f(s)$. The exit threshold itself depends on the values of the other parameters.

In total, we have 8 parameters c_{fa}, c_{fb}, μ_s, ρ, σ_ε^2, μ_G, σ_G^2 and c_e that need to be calibrated. We jointly calibrate these parameters to match a z^* equal to one, 5-year conditional growth rates, 5-year unconditional exit rates, average entrant size, average concentration of entrants, average firm size in 1978, entry rate in 1978 and the average dispersion of log labor productivity for the year 1993 to 2001.

Table 1

		Assigned	
	Value	Definition	Basis
β	0.96	Discount factor	Standard
α	0.64	Curvature	Standard
g	0.01	Labor force growth rate (SS)	Standard

		Jointly Calibrated			
	Parameters		Moments		
	Value	Definition		Data	Model
c_e	8e$-$5	Entry cost	Entry rate 1978	14.75%	14.38%
c_{fa}	3.707	Operating cost intercept	Avg. firm size 1978	20.08	20.37
c_{fb}	0.003	Operating cost slope	SD log-LP 1993-01	0.58	0.61
μ_G	-8.028	Mean of G	Avg. entrant size 1978	5.40	5.32
σ_G^2	3.041	Variance of G	Avg. conc. of entrants	5.90%	5.83%
μ_s	-0.028	Drift in AR(1)	$z^* = 1$	—	—
ρ	0.974	Persistence of AR(1)	5-year growth rate	70.49%	75.26%
σ_ε^2	0.075	Variance of AR(1) shocks	5-year exit rate	48.42 %	53.94%

Some justification for the choice of moments is in order. The cost of entry primarily targets the entry rate in 1978. From the dynamic entry equation, matching the average entrant size in 1978 is necessary to match the entry rate in 1978, so we target this moment. Average entrant size in the model is constant over time. It is determined primarily by μ_G, the mean of the entrant productivity distribution G. The variance σ_G^2 determines the thickness of the right tail of G, and therefore targets the concentration of entrants. The variance of the productivity process σ_ε^2 affects the weight on productivity gridpoints at which firms exit, so it primarily targets the 5-year exit rate. The operating cost intercept c_{fa} plays an important role in determining average firm size. The persistence parameter ρ determines how quickly firms grow, so we use it to target the 5-year growth rate of firms. The operating cost slope c_{fb} plays an important role in matching labor productivity dispersion, so we use it to target the standard deviation of log labor productivity reported in Bartelsman, Haltiwanger and Scarpetta (2013). Table 1 summarizes the parameter values, along with the fit to the targets. The calibrated cost of entry implies that 9.25% of total steady state employment goes towards creation of entrants.

Findings. Figure 4 presents the findings for the entry rate. We highlight three distinct episodes that the model matches well. First, the model generates the steady decline in the entry rate observed between 1978 and 2014. The entry rate in the data declined from 14.5 to 7.9 percent, whereas the entry rate in the model declined from 14.4 to 8.1 percent. Second, the model generates the apparent increase in the entry rate before 1978. This increase is driven by the steady increase in labor force growth during the same time period. The third episode is related to World War II. The years around the war exhibited large fluctuations in the entry rate. The labor force growth series also exhibits large fluctuations around the same time, corresponding to large numbers of civilians leaving the labor force to join the war effort and then returning after the war. Through the lens of our model, these large labor force growth fluctuations translate into similarly large fluctuations in the entry rate. The ability of the calibrated model to match both the long-term trends and short-term fluctuations suggests that changes in labor force growth play a central role in the evolution of the entry rate.

Figure 5 shows how the aggregate exit rate, average firm size and concentration evolve in the model and the data. The model does an excellent job of matching the decline in the aggregate exit rate since 1978. Exit rate declines from 10.5 to 8.4 percent in the model whereas it drops from 10.4 to 7.7 percent in the data. Average firm size increases in both the model and the data. The model, however, overshoots the magnitude of the increase. This occurs because average firm size by age in the model is constant over time, whereas the average size of firms age groups 6-10, 11-15, 16-20, 21-25 and above 25 decline to differing de-

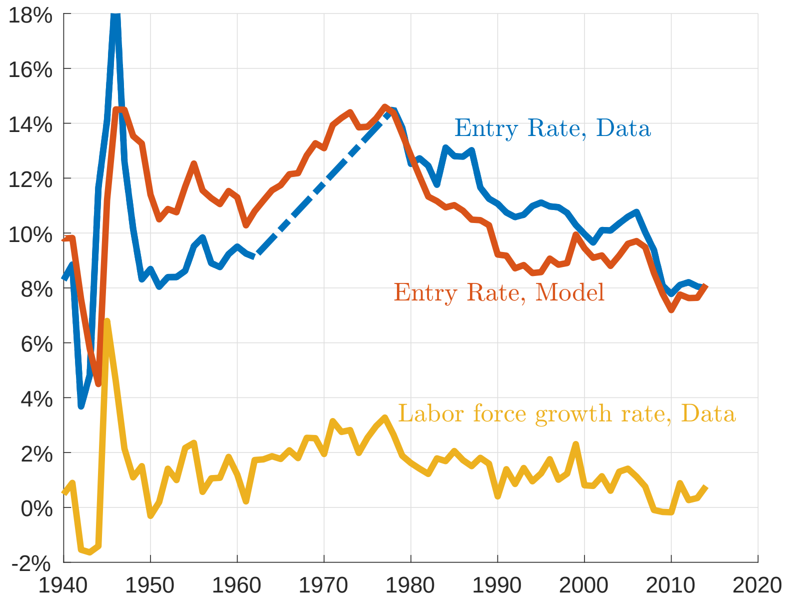


Figure 4

Notes. The entry rate from 1963 to 1977 is linearly interpolated.
Sources. Entry rate 1940-1962: Survey of Current Business. Entry rate 1978-2014: Business Dynamics Statistics. Labor force growth 1940-1946: Lebergott (1964). Labor force growth 1947-2014: Current Population Survey.

grees in the data; see Figure 2.[23] The model also does an excellent job of matching the increase in concentration observed in the data. Starting in 1978, concentration in the model increases from 51.0 to 59.2 percent versus 51.6 to 57.4 percent in the data.

Given that the model does a good job of matching aggregates, it must be the case that model matches firm aging well. Since 1987, the share of 11+ firms in the data increased by 17pp compared to 14pp in the model, and the employment share of firms age 11+ increased by 14pp in the data compared to 11pp in the model.

[23] Average size for these age groupings is relatively constant in the model, with all movements coming from composition effects within the age group. The reason for the decline in size for older age groups in the data is an open question. Current theories include a decline in corporate tax rates (Neira and Singhania, 2019) and an increase in the relative supply of college-educated workers (Ignaszak, 2019). These theories operate through an increase in wages and are thus orthogonal to the mechanism in this paper.

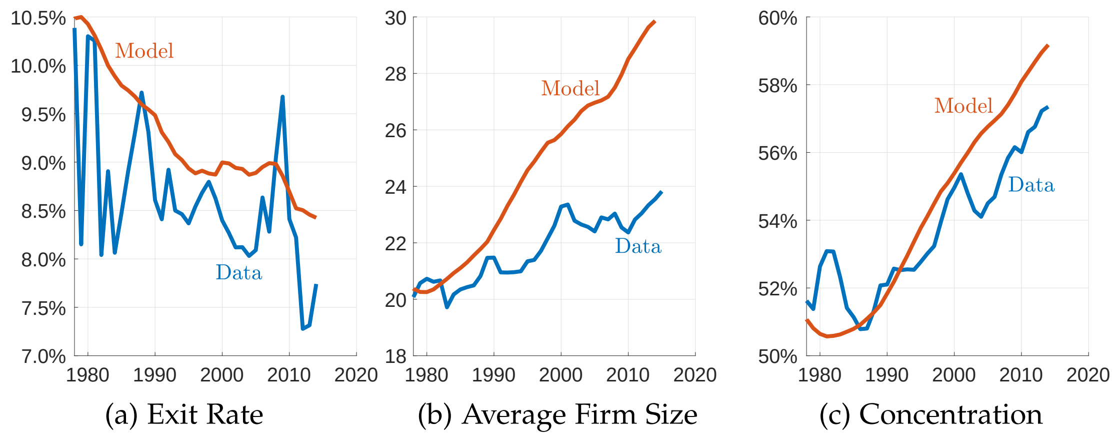


Figure 5

Notes. Concentration is the share of employment in firms with 250+ employees.

4.1 The Role of Feedback Effects of Firm Demographics

In this section we explore the quantitative importance of the feedback effect of firm demographics using identity (1). Table 2 reports the decomposition of the entry rate in the model economy into the direct effect from changes in labor force growth and the feedback effect of firm demographics. The entry rate in the model declines by 6.26 percentage points between 1978 and 2014. Of this decline, 30 percent (1.88pp) is accounted for by the decline in labor force growth. The 30 percent decline would occur even in the absence of firm demographics i.e. without differences in average size and exit rates by firm age. The remaining 70 percent of the decline in the entry rate is due to the feedback effect of firm demographics. The feedback effect can be further broken down into the effect due to a decline in the exit rate (2.05pp), and the effect due to an *increase* in the growth rate of average firm size (1.96pp). The increase in growth rate of average firm size from -1.19pp in 1978 to 0.77pp in 2014 reflects the fact that the time series of average size is U-shaped with its minimum in 1980. The direct effect of labor force growth and the feedback effect together add up to 5.89pp. The remaining 0.37pp is due to a residual arising from changes in labor allocated to the creation of entrants.[24]

[24]When labor is needed for creation of entrants, average firm size in the model is not equal to population divided by the number of firms. In that case identity (1) holds exactly only in a steady state, when the mass of entrants grows at the same rate as population. Along a transition, the accounting identity has an additional term that captures the fact that the growth rate of the mass

Table 2: Entry Rate Decomposition

	Benchmark		
	1978	2014	$\lvert\Delta\rvert$
LF Growth, $\hat{N}$	2.65	0.77	1.88
Exit Rate, ξ	10.48	8.43	2.05
AFS Growth, $\hat{e}$	-1.19	0.77	1.96
Residual	0.06	-0.31	0.37
Entry Rate, λ	14.38	8.12	**6.26**

Notes. All values are in percentage points. With entry costs denominated in units of labor, identity (1) holds approximately, $\lambda \approx \hat{N} + \xi - \hat{e}$. The residual corresponds to changes in the growth rate of labor allocated towards the creation of entrants.

4.2 The Role of Transitional Dynamics

Table 3: Steady State vs. Transitional Effects

	Long-Run Effect			Baby Boom Effect			Adjustment to New Steady State		
	1978 S.S.	2014 S.S.	$\lvert\Delta\rvert$	1978 Bench.	1978 S.S.	$\lvert\Delta\rvert$	2014 S.S.	2014 Bench.	$\lvert\Delta\rvert$
LF Growth, $\hat{N}$	2.65	0.77	1.88	2.65	2.65	0	0.77	0.77	0
Exit Rate, ξ	10.11	9.14	0.97	10.48	10.11	0.37	9.14	8.43	0.71
AFS Growth, $\hat{e}$	0	0	0	-1.19	0	1.19	0	0.77	0.77
Residual	0	0	0	0.06	0	0.06	0	-0.31	0.31
Entry Rate, λ	12.76	9.91	**2.85**	14.38	12.76	**1.62**	9.91	8.12	**1.79**

Notes. All values are in percentage points. The total decline in the entry rate (6.26pp) is the sum of the long-run effect (2.85pp), the baby boom effect (1.62pp), and the adjustment to new steady state effect (1.79pp).

In this section we explore the quantitative importance of transitional dynamics in the decline of the entry rate. Table 3 reports a decomposition of the entry-rate decline in the benchmark into a long-run effect and adjustment effects. The first three columns report the long-run effect. This effect is calculated by comparing the steady state with the 1978 value of labor force growth to the steady state

of entrants, and therefore entry labor, is different from the population growth rate.

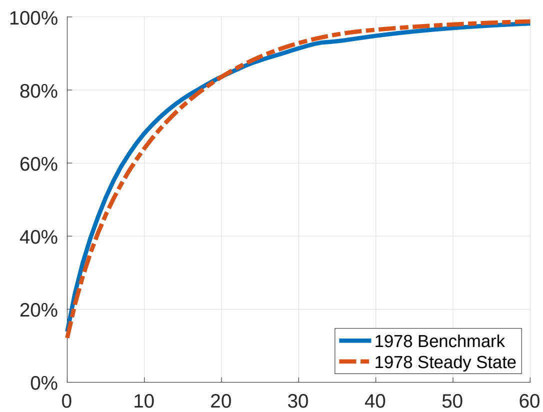
with the 2014 value of labor force growth.[25] The entry rate in the 2014 steady state is 2.85pp lower than the 1978 steady state. This reflects the direct effect of a decline in labor force growth, which accounts for 1.88pp, and the effect of a decline in long-run exit rates, which accounts for 0.97pp. The long-run exit rate declines because lower steady state labor force growth results in an older steady state firm-age distribution. Overall, the long-run effect accounts for 46 percent of the 6.26pp decline in the entry rate.

The adjustment effects account for the remaining 54 percent of the decline in the entry rate. We further break down the adjustment effects into two, one for 1978 and one for 2014. The effect corresponding to each year reflects differences between the transition and the steady state in that year. We refer to the 1978 effect as the baby boom effect because it reflects the effect of the glut of firms born during the expansion of the labor force as a result of the baby boom. We refer to the 2014 effect as the adjustment-to-new-steady-state effect because it reflects the fact that the benchmark model economy is not at a steady state in 2014.

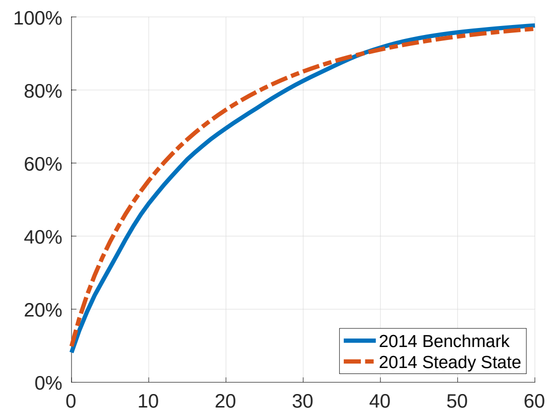
The baby boom effect accounts for a 1.62pp drop in the entry rate. This occurs for three reasons. First, the 1978 exit rate in the benchmark economy is 0.37pp higher than the steady state exit rate. This is because the pre-1978 rise in labor force growth shifts the age distribution towards younger firms — as shown on Figure 6a — which exit at higher rates. Second, the growth in average firm size in the 1978 steady state is zero, while the benchmark exhibits a negative average firm size growth rate of -1.19pp in 1978. Third, a residual of 0.06pp comes from changes in labor allocated to creation of entrants.

The adjustment-to-new-steady-state effect accounts for a 1.79pp drop in the entry rate. This occurs for three reasons. First, firms are older in the 2014 benchmark than in the 2014 steady state — as shown on Figure 6b — reflecting the aging firm distribution that arises as the glut of firms born during the baby boom get older. This implies that the exit rate in the 2014 benchmark is 0.71pp lower than the exit rate in the new steady state. Second, firm aging implies that average firm size is growing during the 2014 transition, which accounts for 0.77pp of the decline in the entry rate. Finally, the residual accounts for the remaining 0.31pp of the decline.

[25]This long-run effect corresponds to the effect in Karahan, Pugsley and Şahin (2018).



(a) 1978 Firm Age Distribution



(b) 2014 Firm Age Distribution

Figure 6

4.3 Entry Rate Projections

The main message from the decomposition exercises is that feedback effects from firm demographics and transitional dynamics are both quantitatively essential for the decline in firm entry. We next explore what projections of labor force growth imply for future entry rates. The Bureau of Labor Statistics (BLS) publishes projections of labor force growth up until the year 2060. We feed the BLS projections into the benchmark model and compute firm entry rates. Figure 7 presents our findings.

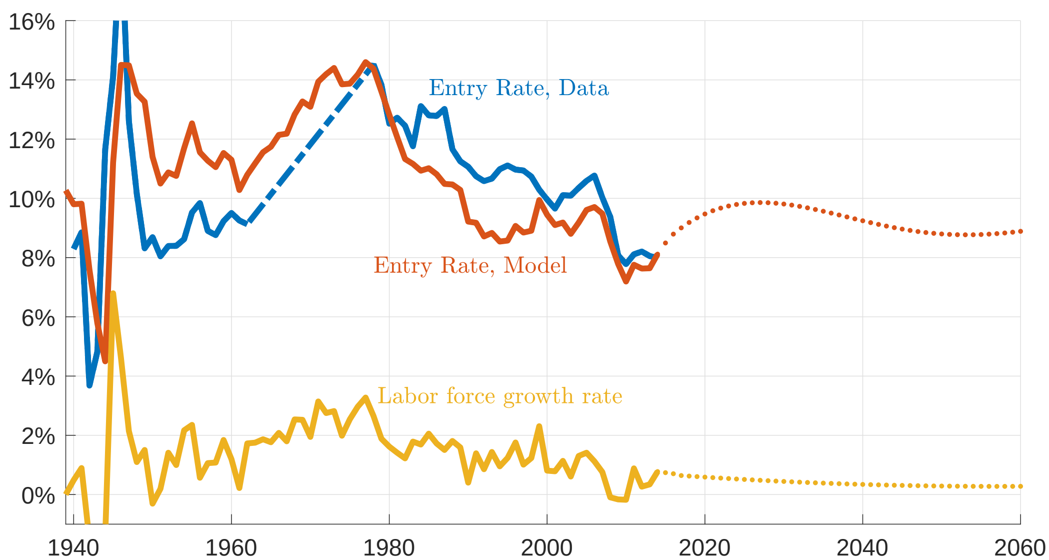


Figure 7: Projections

The BLS projects that the labor force will slowly converge to a growth rate of about 0.25 percent by the year 2060. Through the lens of our model, these projections imply that the entry rate will rise from 8.12 percent in 2014 to 9.12 percent in 2060. The reason for the rebound is threefold. First the exit rate along the transition is lower than the 2060 steady state, 8.43 percent versus 8.84 percent. This is because firms are older along the transition than in the 2060 steady state, and older firms exit at lower rates. Second, average firm size stops growing in the 2060 steady state, which adds an extra 0.77pp to the entry rate. Third the residual goes from 0.31pp to zero as the economy converges to the 2060 steady state. Together these three forces more than offset the 0.52pp decline in the labor force growth rate from 2014 to 2060.

The projections also show that the convergence to the new balanced growth path is non-monotonic. The entry rate rises above and then declines to its stationary level. This cycle in entry rates is due to the dynamic nature of entry. As past entrants age, they grow at slower rates and cannot absorb as much of the growth in labor supply. This creates room for new firms, raising the entry rate. As these new firms age and grow, they absorb a larger fraction of the growth in labor supply, lowering the entry rate and generating firm aging. This cycle repeats and dampens until convergence.

4.4 The Aggregate Labor Share

In this section, we explore quantitatively what firm aging, driven by labor force growth, implies for the recent decline in the aggregate labor share. In recent work, Hartman-Glaser, Lustig and Xiaolan (2019), Kehrig and Vincent (2018) and Autor, Dorn, Katz, Patterson and Van Reenen (2017) document a negative relationship between firm size and labor share. These studies find that almost all of the decline of the aggregate labor share is due to reallocation of value-added from high to low labor share units, rather than a decline in labor share within units. It follows that the decline in the aggregate labor share is primarily due to changes in weights corresponding to the size distribution of firms. Firm aging provides a mechanism that results in such a change in the size distribution. To evaluate the role of firm aging, we need to generate a negative relationship between firm size and labor share. This negative relationship can be generated in various ways without affecting the results. For example, the negative relationship could arise because larger firms produce using technologies that are less labor intensive (Guimaraes and Gil, 2019), have higher markups (Autor, Dorn, Katz, Patterson and Reenen, Forthcoming), or have higher intellectual property products capital (Koh, Santaeulalia-Llopis and Zheng, 2018). To illustrate the aging mechanism, we use the mechanism proposed by Autor, Dorn, Katz, Patterson and Van Reenen (2017) in which labor shares decline with firm size because of overhead labor.

A firm's labor share can be broken down into the share of value added paid to production workers and to overhead labor. In equilibrium, the share paid to production workers is equal to α for all firms. Therefore, all differences in firm-level labor shares are due to the share paid to overhead labor. We have

$$\text{Labor share} = \alpha + \frac{wc_f(s)}{py(s)} = \alpha\left(1 + \frac{c_f(s)}{n(s)}\right) \tag{14}$$

If all firms have the same overhead, $c_f(s) = c_{fa}$, then firm-level labor shares are decreasing in firm size. In our calibration we pick a functional form that allows overhead labor to vary with firm size, $c_f(s)$ equal to $c_{fa} + c_{fb}s^{\frac{1}{1-\alpha}}$. This captures the intuitive idea that larger firms require greater labor overhead. The slope of the overhead function c_{fb} is calibrated to match the standard deviation of log-labor productivity reported in Bartelsman, Haltiwanger and Scarpetta (2013). In spite of requiring higher levels of overhead labor, larger firms in the calibrated model have lower labor shares because the ratio $c_f(s)/n(s)$ declines with firm size. Firm aging reallocates market shares towards older firms, which are larger and have lower labor shares.[26] As a result, the aggregate labor share declines. Figure 8 plots the cumulative change in the aggregate labor share in the model and the data.

We compare the model generated decline to two measures of labor share in the data. First, we take the corporate labor share from 1975-2010 as measured by Karabarbounis and Neiman (2014). Second, we consider an alternative measure of the corporate labor share proposed by Koh, Santaeulalia-Llopis and Zheng

[26] This mechanism is consistent with labor share dynamics in the data: Kehrig and Vincent (2018) document that reallocation occurs towards units that lower their labor share, as opposed to those that have a low level of the labor share.

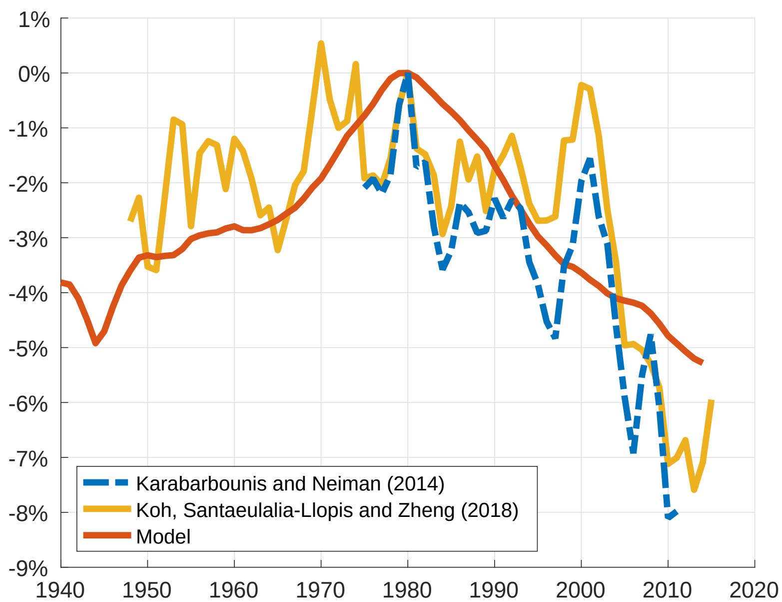


Figure 8: Corporate Labor Share (Relative to 1980)

Notes. For each series, the vertical axis shows the corporate labor share in year t minus its 1980 value.

(2018).[27] The model generates a decline comparable to both the series. The Koh, Santaeulalia-Llopis and Zheng (2018) series exhibit an increase in the labor share from 1947 to 1980, generating an overall hump-shaped aggregate labor share. The model matches this hump-shaped pattern well. The intuition is simple. From 1940 to 1980, the aggregate labor share increases with the entry rate because entrants are small in size, and therefore have higher labor shares. From 1980 onwards, as firms age and grow in size the share of firms with low labor shares increases, leading to a decline in the aggregate labor share.

5 Discussion

Alternative quantitative approach. Table 4 reports how firm demographic variables by age in the model compare to the 1978 to 2014 averages in the data. The calibrated model matches the exit rate well for all ages. The model however,

[27]This measure of the aggregate labor share is different because it accounts for changes in the way the Bureau of Economic Analysis treats intellectual property products. Prior to 1999, intellectual property was treated as a business or consumption expenditure. However, over time the BEA has started treating intellectual property as capital, affecting the measurement of the labor share.

Table 4: Firm Demographic Variables by Age

Age	**Exit rate**		**Average firm size**		**Concentration**	
	Data(%)	Model(%)	Data	Model	Data(%)	Model(%)
0	–	–	6.05	5.32	5.90	5.83
1	21.85	22.24	7.73	6.00	12.29	7.59
2	15.86	15.67	8.46	6.70	13.29	9.25
3	13.43	12.67	9.14	7.49	14.83	10.99
4	11.68	10.90	9.77	8.36	16.45	12.87
5	10.48	9.70	10.36	9.33	17.84	14.97
6-10	8.32	7.85	11.98	12.77	23.00	23.01
11-15	6.40	6.21	15.08	20.86	31.85	38.16
16-20	5.56	5.44	18.81	31.22	40.68	51.31
21-25	4.99	5.01	24.03	42.79	50.47	60.69
Above 25	4.29	4.45	81.59	76.35	78.91	74.36

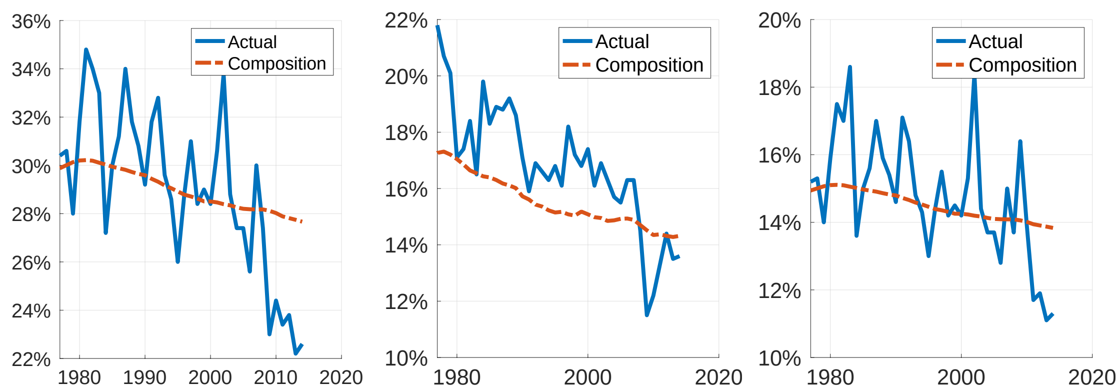
Notes. Concentration is the share of employment in firms with 250+ employees within the age category divided by total employment in the age category.

slightly undershoots average firm size and concentration by age for the 0 to 5 age groups, slightly overshoots these variables for the 11-15, 16-20 and 21-25 age groups, and slightly undershoots these variables for the above 25 group. In order to explore the impact of this difference in age-size profiles on the results, we perform an alternative quantitative approach where we take the age-size profile for 0-25 year olds directly from the data, and impute values for 26-101 year olds using moments of the *Left Censored* firms — the group of firms born before 1977. Because the minimum age of the Left Censored cohort increases every year, these data reveal new information about older ages with each passing year. The details of this exercise are in Appendix C.1. The resulting time series of the entry rate, average size, exit rate and concentration are similar to the benchmark calibration.

Job creation, destruction and reallocation. In addition to firm entry rates, the US has also experienced a decline in the job creation, destruction and reallocation rates.[28] Job creation, destruction and reallocation rates exhibit age effects: older firms create, destroy and reallocate jobs at lower rates. Therefore, firm aging

[28]The job creation rate is the ratio of jobs created, either by entrants or continuers, to total jobs in a period. The job destruction rate is the ratio of jobs destroyed, either by exiting firms or by continuers, to total jobs. The job reallocation rate is equal to creation rate + destruction rate − abs(creation rate − destruction rate).

induced by labor force growth qualitatively generates a decline in aggregate job creation and destruction rates. To explore the quantitative importance of this mechanism, we take average job creation and destruction rates by age from BDS data and use the evolution of the firm-age distribution from the calibrated model to calculate aggregate job creation, destruction and reallocation rates.[29] Figure 9 shows the resulting time series, along with the data. A statistic that summarizes the role of firm aging is the ratio of the slope of the trendline from 1977 to 2014 in the *composition* time series to the slope of the trendline in the *actual* time series for the same years. By this measure, aging explains 47 percent of the decline in the job creation rate, 40 percent of the decline in the job destruction rate, and 35 percent of the decline in the job reallocation rate.[30]



(a) Job Reallocation Rate (b) Job Creation Rate (c) Job Destruction Rate

Figure 9

Productivity. The decline in US entrepreneurship has been linked to declining productivity growth, see e.g. Gourio, Messer and Siemer (2016), Clementi and

[29]Values by age group are the average from the first year the age group is observed to 2006. Value of age groups with multiple ages were assigned to the intermediate age (e.g. the mean of the 6 to 10 age group was assigned to age 8). An 'Above 25' category is created by combining the BDS 26+ age group and the BDS Left-Censored age group. The average of this group was assigned to all ages 31 and older.

[30]The numbers are larger composition effects are calculated using the evolution of the firm age distribution from the alternative quantitative approach in Appendix C.1, or if the trendlines stop in 2006. In the former case, aging explains 64 percent of the decline in the job creation rate, 46 percent of the decline in the job destruction rate, and 40 percent of the decline in the job reallocation rate. In the latter case, aging explains 63 percent of the decline in the job creation rate, 69 percent of the decline in the job destruction rate, and 62 percent of the decline in the job reallocation rate.

Table 5: Average annual productivity growth

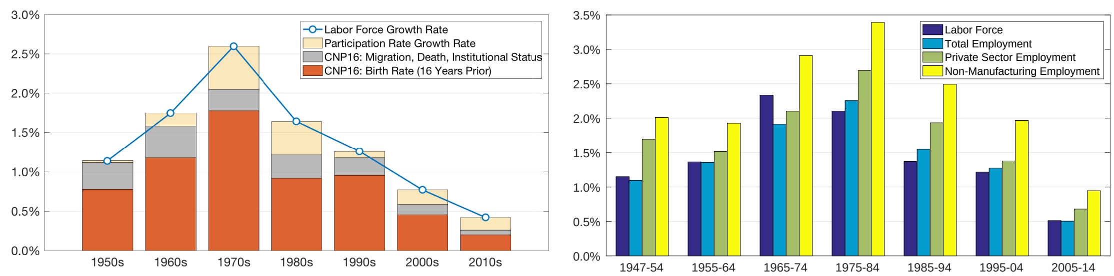
	TFP (%)	Labor productivity (%)
1950s	0.45	−0.004
1960s	0.55	−0.13
1970s	0.83	−0.23
1980s	0.73	0.26
1990s	0.52	0.18
2000s	0.37	0.16
2010s	0.21	0.17

Palazzo (2016), and Decker, Haltiwanger, Jarmin and Miranda (2016). Our model provides a mechanism that links declining entry to declining productivity growth. The aggregate production function in the calibrated model is $TFP \times L^{\alpha}$, where L denotes productive labor and TFP is $(\int s^{1/(1-\alpha)} d\mu(s))^{1-\alpha} M^{1-\alpha}$.[31] The growth rate of TFP in the model depends on changes in the distribution of firm productivities and growth in the number of firms, both of which are affected by firm entry. Therefore, changes in entry induced by changes in population growth also affect productivity growth. Table 5 shows that average annual TFP growth by decade in the model inherits the hump shape of labor force growth and declines concurrently with firm entry rates. The table also shows average annual growth rate of labor productivity by decade. Entrants have low levels of labor productivity, but grow at a faster rate than incumbents, so the effect of higher entry on labor productivity growth is ambiguous. Quantitatively, the level effect dominates: labor productivity shrinks in decades of increasing entry rates, from the 1950s to the 1970s, and grows in decades of declining entry.

Labor force. What are the main drivers of labor force growth? Figure 10a plots the labor force growth rate by decade, dividing the bars into the percentage contribution of growth in participation rates, birth rates sixteen years prior, and other. The contribution of participation rate growth in labor force growth is small de-

[31] Because it does not treat firms as a factor of production, this measure is sometimes referred to as *measured TFP*. Alternatively, TFP can be calculated as the residual when both firms and labor are treated as factors of production, $(\int s^{1/(1-\alpha)} d\mu(s))^{1-\alpha}$. This alternative measure of TFP follows a pattern similar to labor productivity.

spite the increase in female labor force participation since the 1950s. The reason is that male labor force participation declined over the same time period, dampening the effect of female labor force participation on total labor force participation. The bulk of the changes in labor force growth are accounted for by birth rates sixteen years prior.[32]



(a) Sources

(b) Alternative Measures

Figure 10: Labor Force Growth

Sources. Current Population Survey and Current Employment Statistics.
Notes. Panel A: Details on Appendix A.2. Panel B: Units are average annual growth rates. Data starts in 1947. Decade cutoffs are chosen so that full business cycles fall within the decade bin, effectively capturing the trend component in growth rates.

One potential source of concern when using the civilian labor force as a measure of labor supply is that it includes the unemployed, those employed by government, and the self-employed. Figure 10b shows that total employment growth (excludes the unemployed) and private sector employment growth (excludes the self-employed and those working for government) follow a similar hump-shaped pattern as labor force growth. Another potential source of concern is the manufacturing sector, which has experienced negative overall employment growth since the 1980s (Fort, Pierce and Schott, 2018). An exodus of workers from manufacturing into non-manufacturing could reverse the trend of declining employment growth in non-manufacturing sectors. Figure 10b shows that this is not the case. Non-manufacturing employment growth also shows a similar rise and fall pattern as labor force growth. The decline of manufacturing employment does not

[32]Details of the decomposition are on Appendix A.2. On average, birth rates sixteen years prior account for 64 percent of labor force growth across decades. The actual contribution of the birth rate to labor force growth is higher than 64 percent because the birth rate also has an effect on participation rates. For example, a portion of the decline of participation rates since the year 2000 is due to the baby boomer generation reaching the age of 55 and over, whose age group has low participation rates.

have a large effect on nonmanufacturing employment growth partly because the flow of workers out of manufacturing is small compared to the flow of workers entering the labor force. From 1977 to 2014, manufacturing employment shrank by 6 million workers while the labor force grew by 57 million workers.

6 Final Remarks

Recent decades have witnessed a decline in firm entry and exit rates, and an increase in employment concentration and average firm size. In contrast, none of these trends appear within firm-age bins. Therefore, the bulk of the aggregate change is explained by the aging of firms as a result of the decline in firm entry. The interplay of population growth and firm demographics can generate both the stability within firm-age bins and the aggregate behavior of these variables. While the direct effect of population demographics on the creation of new firms accounts for one-third of the total effect, the feedback from firm demographics accounts for the remaining two-thirds. Transitional dynamics play a key role within these feedback effects, accounting for half of the total change. Given a negative relationship between firm size and labor shares, firm aging induced by population growth also replicates the hump-shaped pattern of the aggregate labor share.

As pointed out in the theory section, our analysis is consistent with various kinds of models of perfect and imperfect competition. Without further details, it is not possible to discuss policy recommendations. In the case of perfect competition, standard welfare theorems apply so the equilibrium is Pareto Optimal. In contrast, the rise in markups in our third example could have negative welfare implications as the share (not size) of high markup firms increases.[33] Similar considerations can be made about the fall in entry. While the reduction in the number of firms could have resulted in a fall in measured productivity, as pointed out in the paper, the policy implications again are not obvious. In our competitive benchmark, this fall in entry is an optimal response to the decrease in population growth and policies aimed at mitigating this effect would have distorted extensive

[33]Using a model with variable markups, Edmond, Midrigan and Xu (2018) find sizable costs of aggregate markups. However, as in Baqaee and Farhi (2019), they find that the observed rise in aggregate markups could have resulted in lower welfare costs as a result of the decrease in markup dispersion.

vs. intensive margins in the allocation of resources. In contrast, in a model where entrants have positive external effects (e.g. Luttmer (2007)) such kind of policies could be justified. More generally, if firms are considered knowledge, such policies might have a role in models of endogenous technical change based on Romer (1990).[34]

The role of demographics in economics has been receiving increased attention as a result of the sharp decreases in fertility and consequent aging of the population observed in most developed economies during the last half of the century. This research suggests that demographic trends have played a sizable role in explaining some of the recent macro trends. In this paper we have singled out the role of demographic changes as a driving force for the decrease in entry and pointed out the importance of the feedback coming from firm demographics. Several authors have emphasized the role of other potential factors, such as changes in economies of scale or the incentives for innovation. While these forces are likely to have contributed to explaining the observed trends in the evolution of aggregate productivity and firm dynamics, our analysis underscores that the effect of changes in population growth on firm dynamics are first-order, and therefore should not be ignored.

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Online appendices

Appendix A

A.1 Data Appendix

Civilian Labor Force Growth Rate 1940-2014. Civilian labor force data comes from the Bureau of Labor Statistics (BLS) Current Population Survey for the years 1947 to 2014, and from Lebergott (1964) from 1940 to 1946. The civilian labor force definition in BLS includes population 16 years of age and over while the definition in Lebergott includes population 14 years of age and over. We check the comparability of the two series from 1947 to 1960, the years where the two series overlap. Labor force growth rates of ages 14+ and 16+ are nearly identical for these years.

Firm Data 1978-2014. Firm data comes from the U.S. Census Bureau's Business Dynamics Statistics (BDS). The BDS dataset has near universal coverage of private sector firms with paid employees. BDS data starts in 1977, but common practice suggests dropping 1977 and 1978 due to suspected measurement error (e.g. Moscarini and Postel-Vinay, 2012). We drop entry rates for 1977, but keep 1978, as calibrating to 1978 or 1979 does not affect our quantitative results (the model matches the entry rate in both 1978 and 1979 almost exactly). We exclude the first four years of BDS data for age groups 6-10, 11-15, 15-20, and 20-25 in order to have consistent age-group definitions.

Firm Entry Rates 1940-1962. The firm entry rate is obtained from the now-discontinued U.S. Department of Commerce's Survey of Current Business. This dataset includes all nonfarm businesses, including firms with zero employees. The entry rate is defined as 'New Businesses' divided by 'Operating Businesses'. The 1963 edition was the last one to report a 'Business Population and Turnover' section. From 1963, the Survey of Current Business reported 'Business Incorporations' instead, which only include stock corporations.

Birth Rates. The 1930 to 2000 birth rate series is from the CDC National Center for Health Statistics.

Employment 1947-2014. Total employment corresponds to the civilian employment in the BLS Current Population Survey. Private sector employment and manufacturing employment are from the BLS Current Employment Statistics (Establishment Survey). Non-manufacturing employment is private sector employment minus manufacturing employment.

Labor Force Projections. Projections of labor force growth are from the BLS; see Toossi (2016).

A.2 Figure 10a: Labor Force Decomposition

The BLS' definition of labor force is

$$\mathrm{LF}_t = \mathrm{CNP16}_t \times \mathrm{PR}_t,$$

where LF_t is the civilian labor force at time t, $\mathrm{CNP16}_t$ is the civilian noninstitutional population age 16 and over at time t, and PR_t is the participation rate at time t. It follows that labor force growth rate is the sum of the growth rate of each component,

$$\text{LF Growth Rate}_t = \text{CNP16 Growth Rate}_t + \text{PR Growth Rate}_t.$$

We further decompose CNP16 growth rate at time t into the birth rate at time $t-16$ and a residual term Other_t,

$$\text{CNP16 Growth Rate}_t = \text{Birth Rate}_{t-16} + \text{Other}_t,$$

where the Other_t term includes death rates, net migration rates, and rates of entry and exit into institutional status. The size of the bars is calculated as the absolute value of the growth rate of a component divided by the sum of absolute growth rates of each component, multiplied by labor force growth. For example, the size of the PR16 GR bar in the 2010s is 0.15 percent. This is the PR16 GR (-0.6 percent) in absolute terms divided by the sum of CNP16 GR (1 percent) plus the PR16 GR in absolute terms, multiplied by labor force growth (0.42 percent).

Appendix B Firm Age Regressions

Table B-1: Regression of concentration on year

Panel A. (Size 250+)			Panel B.		
Variable	Specification		Measure	Specification	
	(1)	(2)		(1)	(2)
Year	0.003***	−0.000			
	(0.001)	(0.000)			
AGE:			SIZE:		
Age 0		0.064***	**250+**	0.003***	−0.000
		(0.007)	R^2	0.027	0.983
Age 1		0.128***	Observations	345	307
		(0.007)			
Age 2		0.138***	**500+**	0.003***	−0.000
		(0.007)	R^2	0.022	0.979
Age 3		0.154***	Observations	345	307
		(0.007)			
Age 4		0.170***	**1000+**	0.002**	−0.000**
		(0.007)	R^2	0.016	0.977
Age 5		0.184***	Observations	343	305
		(0.008)			
Age 6 to 10		0.236***	**2500+**	0.002**	−0.000***
		(0.008)	R^2	0.014	0.982
Age 11 to 15		0.325***	Observations	320	282
		(0.009)			
Age 16 to 20		0.412***	**5000+**	0.002*	−0.000***
		(0.010)	R^2	0.011	0.986
Age 21 to 25		0.501***	Observations	273	236
		(0.011)			
Age Above 25		0.783***	**10000+**	0.001	−0.000***
		(0.012)	R^2	0.008	0.988
			Observations	201	164
R^2	0.027	0.983			
Observations	345	307			

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Notes. In both panels, specification (1) reports that the trend in concentration is positive. Specification (2) reports that the positive trend disappears with age controls. In Panel A, concentration is measured as share of employment in firms with 250+ employees. Panel B shows the coefficient on year for different concentration thresholds. The standard error of coefficients for all measures in Panel B is identical, equal to 0.001 and 0.000 respectively, so they are omitted from the table.

Table B-2: Regression of log average firm size on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	0.009^{***}	-0.005^{***}	-0.005^{***}	-0.005^{***}
	(0.001)	(0.001)	(0.000)	(0.000)
AGE:				
Age 0		1.842^{***}	1.416^{***}	1.463^{***}
		(0.024)	(0.016)	(0.024)
Age 1		2.087^{***}	1.662^{***}	1.721^{***}
		(0.024)	(0.016)	(0.025)
Age 2		2.177^{***}	1.752^{***}	1.811^{***}
		(0.025)	(0.016)	(0.025)
Age 3		2.253^{***}	1.828^{***}	1.873^{***}
		(0.025)	(0.017)	(0.026)
Age 4		2.324^{***}	1.899^{***}	1.946^{***}
		(0.026)	(0.017)	(0.026)
Age 5		2.384^{***}	1.959^{***}	2.008^{***}
		(0.026)	(0.017)	(0.026)
Age 6 to 10		2.531^{***}	2.106^{***}	2.164^{***}
		(0.029)	(0.018)	(0.029)
Age 11 to 15		2.753^{***}	2.328^{***}	2.334^{***}
		(0.032)	(0.019)	(0.032)
Age 16 to 20		2.976^{***}	2.551^{***}	2.497^{***}
		(0.036)	(0.021)	(0.036)
Age 21 to 25		3.215^{***}	2.790^{***}	2.610^{***}
		(0.041)	(0.023)	(0.042)
Age Above 25		4.379^{***}	3.954^{***}	3.170^{***}
		(0.042)	(0.024)	(0.043)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR×AGE CONTROLS	No	No	No	Yes
Observations	3,105	2,763	2,763	2,763
R^2	0.014	0.977	0.994	0.996

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Notes. Specification (1) reports that the trend in log-average firm size is positive. Specification (2) reports that the positive trend disappears with age controls. Specifications (3) and (4) control for sector and sector-age effects.

Table B-3: Regression of exit rate on year

Variable	Specification			
	(1)	(2)	(3)	(4)
Year	-0.148^{***}	-0.014^{**}	-0.014^{***}	-0.014^{***}
	(0.011)	(0.006)	(0.005)	(0.005)
AGE:				
Age 1		21.963^{***}	19.656^{***}	19.301^{***}
		(0.181)	(0.190)	(0.338)
Age 2		16.224^{***}	13.916^{***}	12.782^{***}
		(0.184)	(0.192)	(0.343)
Age 3		13.755^{***}	11.448^{***}	10.846^{***}
		(0.188)	(0.194)	(0.348)
Age 4		12.112^{***}	9.805^{***}	9.463^{***}
		(0.191)	(0.196)	(0.353)
Age 5		10.838^{***}	8.531^{***}	8.416^{***}
		(0.195)	(0.199)	(0.359)
Age 6 to 10		8.739^{***}	6.432^{***}	6.787^{***}
		(0.214)	(0.211)	(0.390)
Age 11 to 15		6.807^{***}	4.500^{***}	5.273^{***}
		(0.236)	(0.226)	(0.431)
Age 16 to 20		6.015^{***}	3.708^{***}	4.762^{***}
		(0.264)	(0.245)	(0.486)
Age 21 to 25		5.478^{***}	3.171^{***}	4.496^{***}
		(0.301)	(0.272)	(0.569)
Age Above 25		4.691^{***}	2.384^{***}	4.591^{***}
		(0.311)	(0.279)	(0.591)
SECTOR CONTROLS	No	No	Yes	Yes
SECTOR$\times$AGE CONTROLS	No	No	No	Yes
Observations	2,763	2,421	2,421	2,421
R^2	0.064	0.962	0.975	0.978

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Notes. Specification (1) reports that the trend in the exit rate is negative. Specification (2) reports that the negative trend nearly disappears with age controls. Specifications (3) and (4) control for sector and sector-age effects.

Appendix C

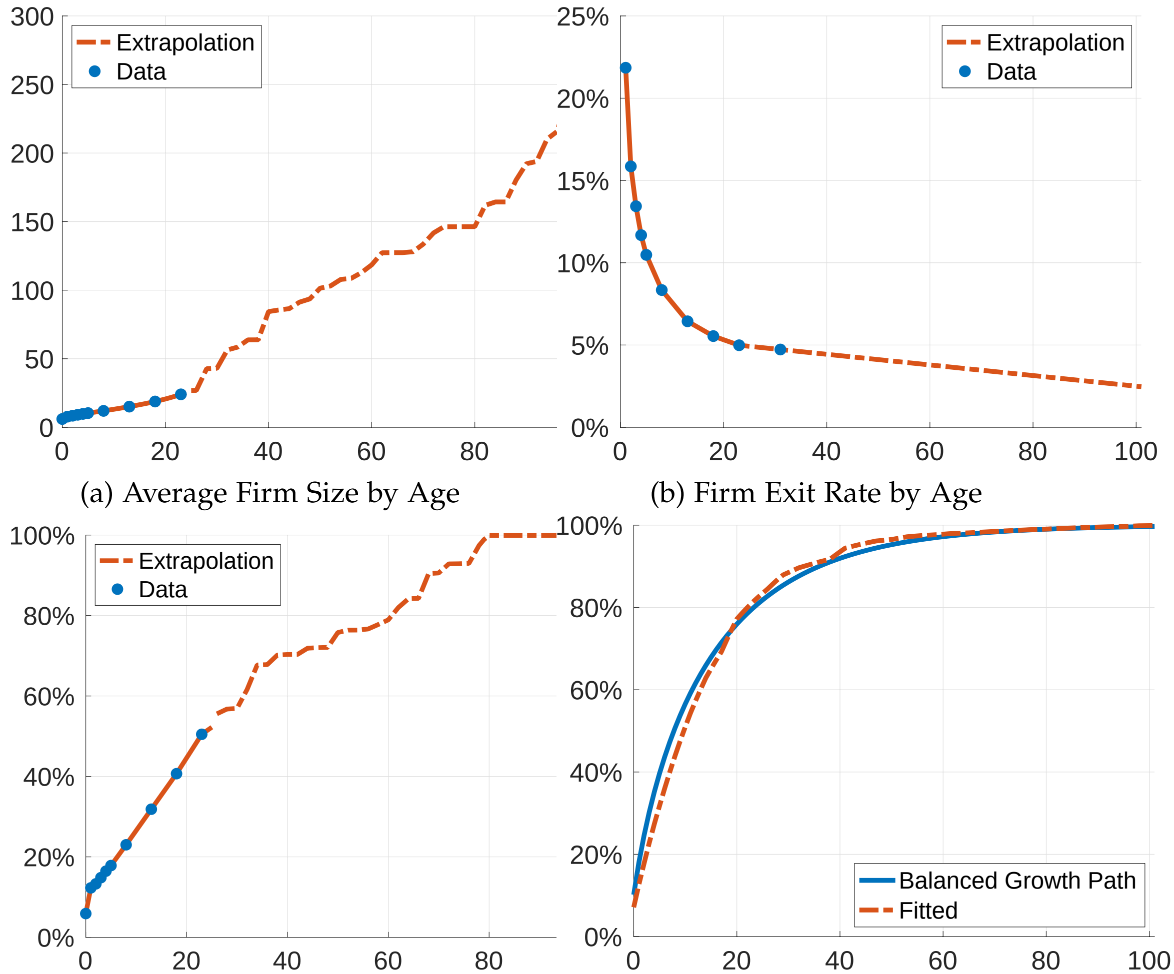
C.1 Alternative Quantitative Approach

This approach uses a non-parametric method to impute (i) firm demographic variables for older firms and (ii) the 1940 firm-age distribution from data on Left Censored firms. By the Time Invariance Corollary, we can infer exit rates, average firm size and concentration by age for 1940 to 2014 from sample averages of the observed levels. This strategy applies directly for ages 0 to 5. For ages 6-25, the BDS provides this data in five-year bins (e.g. 6-10). We linearly interpolate the firm demographic variables for these intermediate ages by assigning the group average value to the median age of the group (i.e. we assign the average firm size of the 6-10 group to age 8). Ages 26-101 are divided into 38 bins, with the last bin corresponding to ages greater than or equal to 101. The value of average firm size and concentration in these 38 bins is set to match the 38 years of the corresponding time series of the Left Censored group. We extrapolate firm exit rates for ages 26-101 linearly by using the trendline that runs through the average exit rate of the 21-25 age group and the 26-37 age group.[35] The firm-age employment distribution in 1940 is obtained by partitioning the age grid into 38 age-weight bins and picking age-weights that match the employment weight of the Left Censored group for the 38 years of data. The dynamic entry equation requires two more parameters, S_0 and c_e. The ratio of these parameters determines the volatility of the entry rate time series. We normalize c_e to unity and calibrate S_0 to match the volatility of the entry rate times series.

The algorithm proceeds in two steps. Given linearly interpolated exit rates by age, we guess a firm-age distribution in 1940 and average firm size for ages 26-101. We then feed labor force growth data from 1940 to 2014 through the dynamic entry equation to calculate the evolution of the firm-age distribution, and compare the fit of the average firm size and the employment weight time series of the Left Censored cohort in the simulation and in the data. The initial guess is then updated using a Nelder-Mead simplex algorithm (imposing penalties for non-monotonic updates) and iterates until the distance between the 76 simulated moments and data cannot be reduced any further. Distance is measured as the square root of the sum of squared log-differences. The second step takes the values for average firm size and exit rate by age and the initial distribution from the first step as given and determines concentration levels for ages 26-101 to match the concentration time series of the Left Censored group.

Figures C-1 presents the extrapolated values of average size, exit rate and concentration by age. Figure C-1d shows that the resulting 1940 distribution is similar to the age-distribution along the balanced growth path in the second approach. Figure C-2 shows the match of our extrapolation strategy to the time series of average firm size, exit rate, concentration and employment shares of Left Censored firms. Figure C-3 shows the implied evolution of entry rates, average firm size, exit rates and concentration.

[35] The mean of the 26-37 age group is the average from 2003 to 2014 of the group labeled 26+ in BDS.

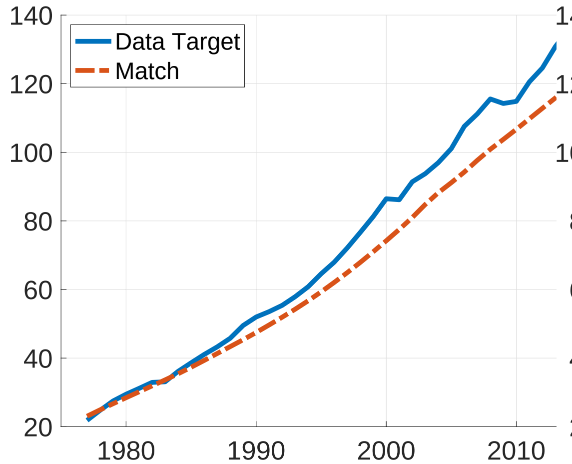


(a) Average Firm Size by Age (b) Firm Exit Rate by Age

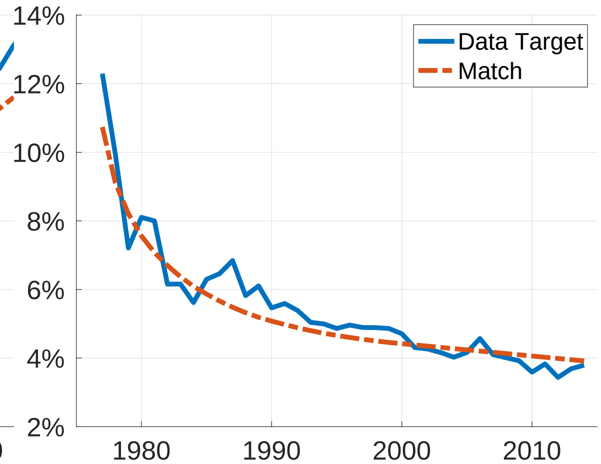
(c) Firm Concentration by Age (d) Firm Age Distribution in 1940

Figure C-1

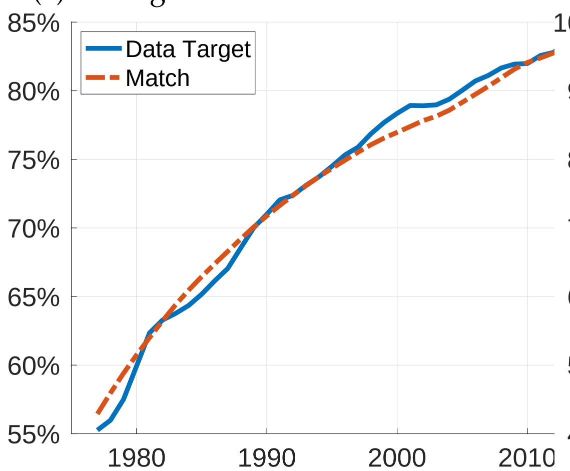
Notes. Dots are the sample average for the age group. Value of age groups with multiple ages were assigned to the median age (e.g. the mean of the 6 to 10 age group was assigned to age 8). Values in between dots are interpolated. Dashed lines are extrapolations set to match moments of the left-censored cohort in Figure C-2. Figure C-1d compares the initial distribution used in the first approach exercise (fitted) with a balanced growth path distribution (with labor force growth of one percent).



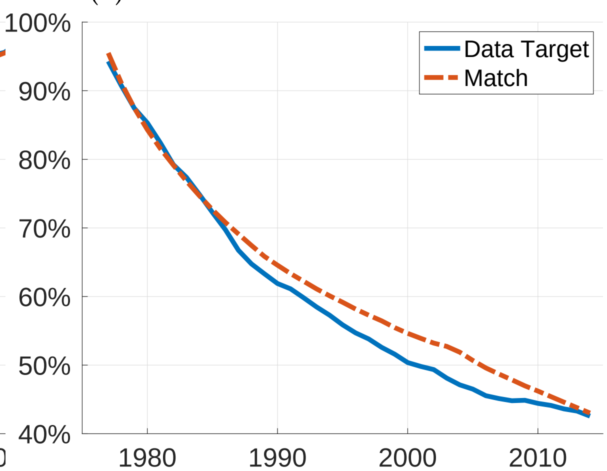
(a) Average Firm Size Left Censored



(b) Exit Rate Left Censored



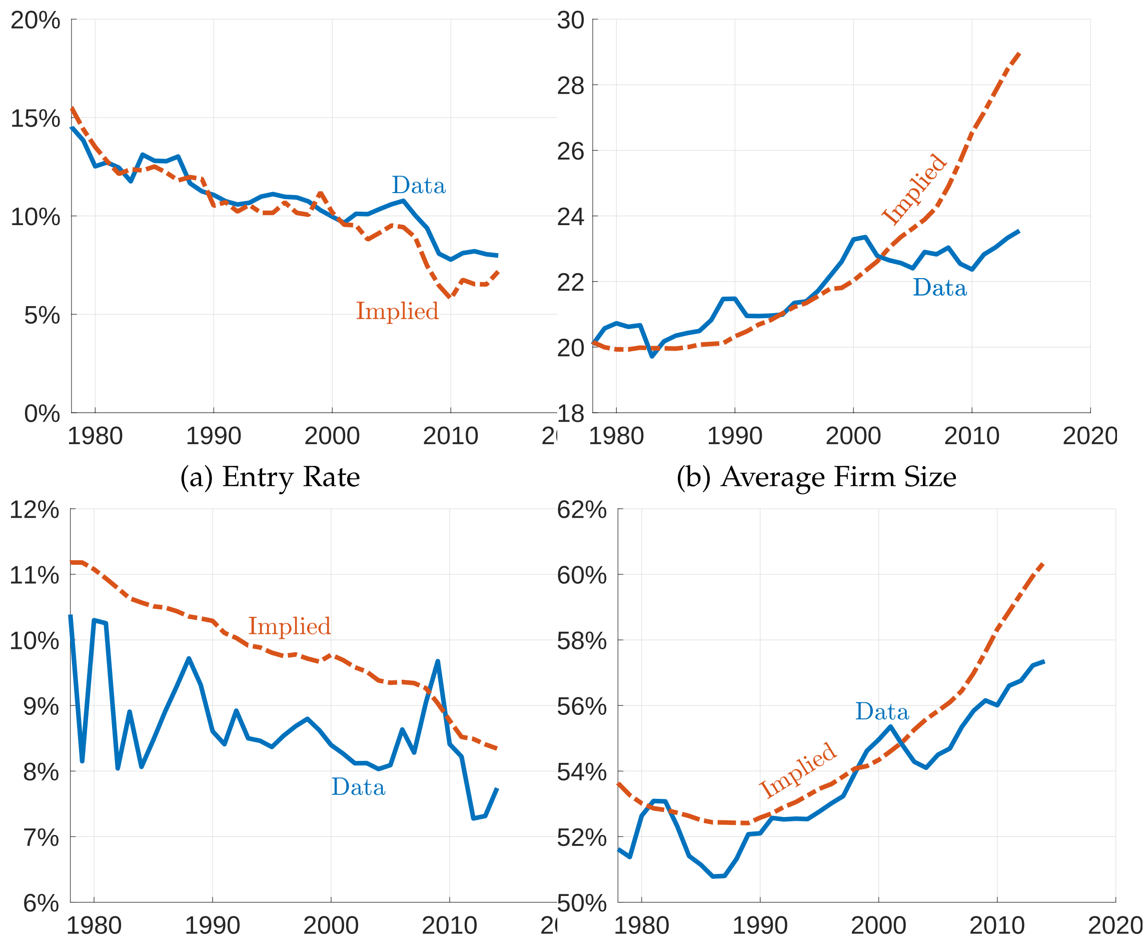
(c) Concentration Left Censored



(d) Employment Weight Left Censored

Figure C-2: Moments of the Left Censored cohort

Notes. Left censored firms are those born before 1977.



(a) Entry Rate

(b) Average Firm Size

(c) Exit Rate

(d) Concentration

Figure C-3

Appendix D Mapping to variable markup economy

In this section we discuss how one might generate the calibrated employment process in the variable markup economy. The key elements are $p(s), G(s)$ and the transition function $F(s'|s)$. Employment is given by

$$n(s,z) \quad = \quad z(s/p(s))^{\sigma}. \tag{A-1}$$

and profits

$$\pi(s,z) = z(s/p(s))^{\sigma}(p(s)-1) - c_f \tag{A-2}$$

For given σ and z, there is a one-one correspondence between $s/p(s)$ and the firm state in the baseline model. In particular, $s^*/p(s^*)$ is pinned down by the employment threshold, the distribution of employment of entrants pins down an initial distribution for the variable $s/p(s)$ on

$s \geq s^*$, and the Markov process for employment pins down its conditional distribution. Likewise, we can map the fixed cost function in the baseline model into a fixed cost function $c_f\left(s/p\left(s\right)\right)$ in this variable markups setting.

We can write the value function:

$$v\left(s,z\right) = \pi\left(s,z\right) + \max\left(0, Ev\left(s',z\right)|s\right)$$

where upon substitution

$$\pi\left(s,z\right) = n\left(s,z\right)\left(p\left(s\right) - 1\right)$$

The value function has to meet two conditions:

$$v\left(s^*,z\right) = 0$$

and $\int v\left(s,z\right)dG\left(s\right) = c_e$, where the value c_e is the same as in the baseline model. Our previous analysis puts restrictions only on $s/p\left(s\right)$, and for fixed values of $s/p\left(s\right)$ profits are increasing in $p\left(s\right)$. These two conditions can be easily met, given the flexibility in the choice of the function $p\left(s\right)$.